OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

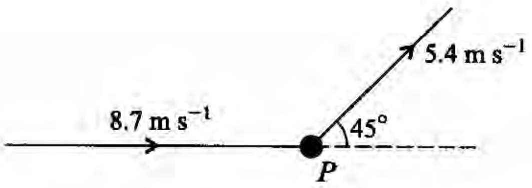
- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1



A particle P of mass 0.4 kg moving in a straight line has speed 8.7 m s^{-1}. An impulse applied to P deflects it through 45° and reduces its speed to 5.4 m s^{-1} (see diagram). Calculate the magnitude and direction of the impulse exerted on P. [7]

2 O is a fixed point on a horizontal straight line. A particle P of mass 0.5 kg is released from rest at O. At time t seconds after release the only force acting on P has magnitude $(1 + kt^2)$ N and acts horizontally and away from O along the line, where k is a positive constant.

(i) Find the speed of P in terms of k and t. [3]

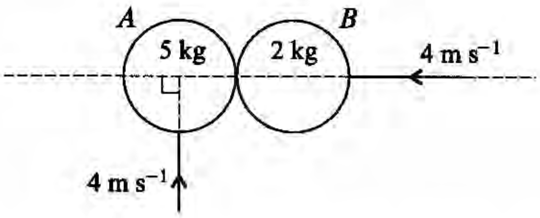
(ii) Given that P is 2 m from O when $t = 1$, find the value of k and the time taken by P to travel 20 m from O. [5]

3 A light elastic string has natural length 3 m. One end is attached to a fixed point O and the other end is attached to a particle of mass 1.6 kg. The particle is released from rest in a position 5 m vertically below O. Air resistance may be neglected.

(i) Given that in the subsequent motion the particle just reaches O, show that the modulus of elasticity of the string is 117.6 N. [4]

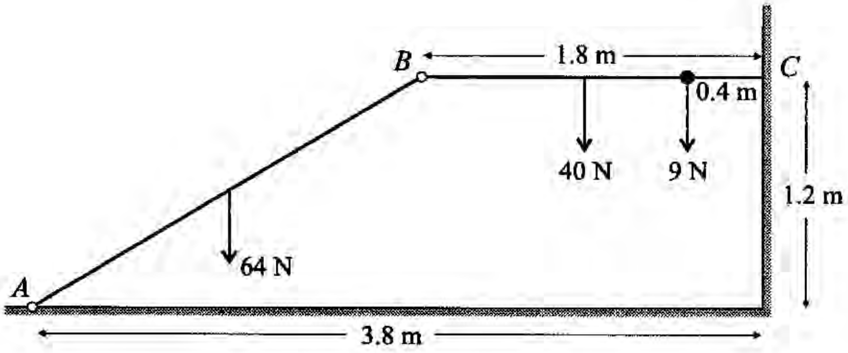
(ii) Calculate the speed of the particle when it is 4.5 m below O. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 5 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed 4 m s^{-1} and is moving perpendicular to the line of centres, and B has speed 4 m s^{-1} along the line of centres (see diagram). The coefficient of restitution is 0.75. Find the speed and direction of motion of each sphere immediately after the collision. [10]

5



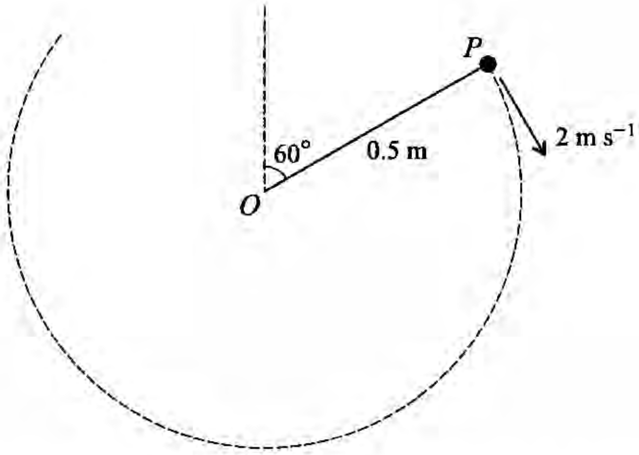
Two uniform rods AB and BC have weights 64 N and 40 N respectively. The rods are freely jointed to each other at B. The rod AB is freely jointed to a fixed point on horizontal ground at A and the rod BC rests against a vertical wall at C. The rod BC is 1.8 m long and is horizontal. A particle of weight 9 N is attached to the rod BC at the point 0.4 m from C. The point A is 1.2 m below the level of BC and 3.8 m from the wall (see diagram). The system is in equilibrium.

(i) Show that the magnitude of the frictional force at C is 27 N. [4]

(ii) Calculate the horizontal and vertical components of the force exerted on AB at B. [5]

(iii) Given that friction is limiting at C, find the coefficient of friction between the rod BC and the wall. [2]

6



One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. With the string taut and at an angle of 60° to the upward vertical, P is projected with speed $2\,\mathrm{m\,s^{-1}}$ (see diagram). P begins to move without air resistance in a vertical circle with centre O. When the string makes an angle θ with the upward vertical, the speed of P is $v\,\mathrm{m\,s^{-1}}$.

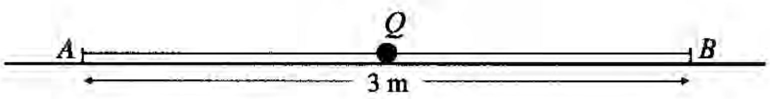
(i) Show that $v^2 = 8.9 - 9.8\cos\theta$. [4]

(ii) Find the tension in the string in terms of θ. [4]

(iii) P does not move in a complete circle. Calculate the angle through which OP turns before P leaves the circular path. [4]

[Turn over

7



As shown in the diagram, A and B are fixed points on a smooth horizontal table, where $AB = 3$ m. A particle Q of mass 1.2 kg is attached to A by a light elastic string of natural length 1 m and modulus of elasticity 180 N. Q is attached to B by a light elastic string of natural length 1.2 m and modulus of elasticity 360 N.

(i) Verify that when Q is in equilibrium $BQ = 1.5$ m. [4]

Q is projected towards B from the equilibrium position with speed $u\,\text{m s}^{-1}$. Subsequently Q oscillates with simple harmonic motion.

(ii) Show that the period of the motion is 0.314 s approximately. [5]

(iii) Show that $u \leqslant 6$. [4]

(iv) Given that $u = 6$, find the time taken for Q to move from the equilibrium position to a position 1.3 m from A for the first time. [3]

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4730

Mechanics 3

Monday **22 MAY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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[Turn over

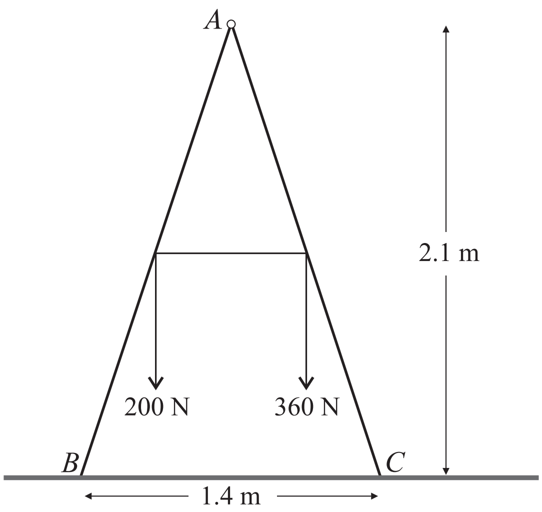
1 A ball of mass 0.4 kg is moving in a straight line, with speed $25\,\mathrm{m\,s^{-1}}$, when it is struck by a bat. The bat exerts an impulse of magnitude 20 N s and the ball is deflected through an angle of 90°. Calculate

(i) the direction of the impulse, [3]

(ii) the speed of the ball immediately after it is struck. [3]

2 A duck of mass 2 kg is travelling with horizontal speed $4\,\mathrm{m\,s^{-1}}$ when it lands on a lake. The duck is brought to rest by the action of resistive forces, acting in the direction opposite to the duck's motion and having total magnitude $(2v + 3v^2)$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the duck. Show that the duck comes to rest after travelling approximately 1.30 m from the point of its initial contact with the surface of the lake. [8]

3



Two uniform rods AB and AC, of equal lengths, and of weights 200 N and 360 N respectively, are freely jointed at A. The mid-points of the rods are joined by a taut light inextensible string. The rods are in equilibrium in a vertical plane with B and C in contact with a smooth horizontal surface. The point A is 2.1 m above the surface and $BC = 1.4$ m (see diagram).

(i) Show that the force exerted on AB at B has magnitude 240 N and find the tension in the string. [6]

(ii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. [3]

4 A particle is connected to a fixed point by a light inextensible string of length 2.45 m to make a simple pendulum. The particle is released from rest with the string taut and inclined at 0.1 radians to the downward vertical.

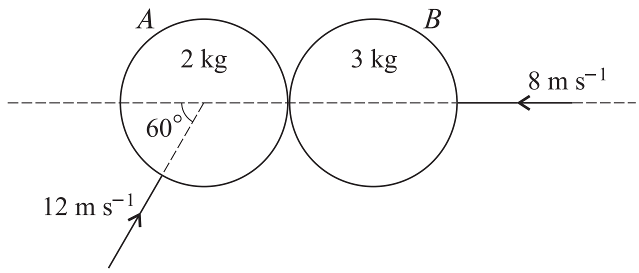
(i) Show that the motion of the particle is approximately simple harmonic with period 3.14 s, correct to 3 significant figures. [5]

Calculate, in either order,

(ii) the angular speed of the pendulum when it has moved 0.04 radians from the **initial** position, [3]

(iii) the time taken by the pendulum to move 0.04 radians from the **initial** position. [3]

5



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision A is moving with speed $12\,\text{m}\,\text{s}^{-1}$ at 60° to the line of centres, and B is moving with speed $8\,\text{m}\,\text{s}^{-1}$ along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [12]

6 A bungee jumper of mass 70 kg is joined to a fixed point O by a light elastic rope of natural length 30 m and modulus of elasticity 1470 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Find the distance fallen by the jumper when maximum speed is reached. [4]

(ii) Show that this maximum speed is $26.9\,\text{m}\,\text{s}^{-1}$, correct to 3 significant figures. [4]

(iii) Find the extension of the rope when the jumper is at the lowest position. [4]

[Question 7 is printed overleaf.]

7

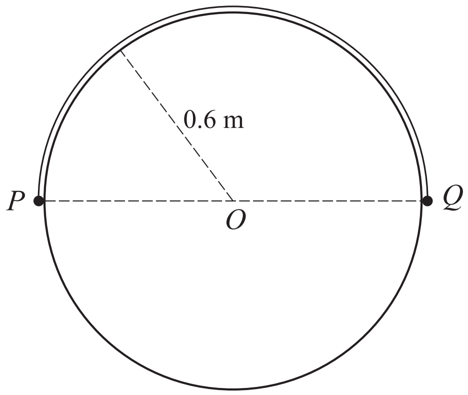


Fig. 1

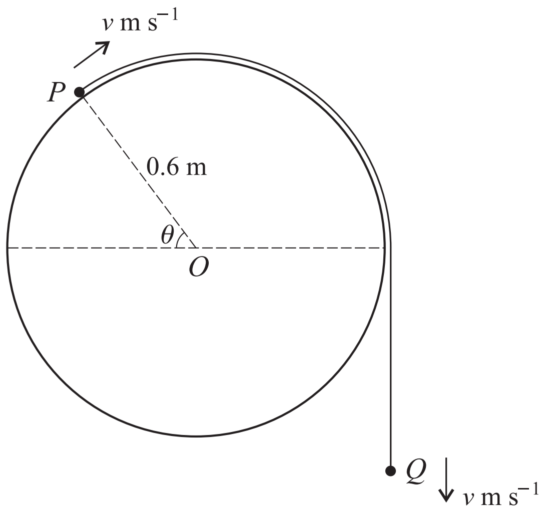


Fig. 2

A smooth horizontal cylinder of radius 0.6 m is fixed with its axis horizontal and passing through a fixed point O. A light inextensible string of length 0.6π m has particles P and Q, of masses 0.3 kg and 0.4 kg respectively, attached at its ends. The string passes over the cylinder and is held at rest with P, O and Q in a straight horizontal line (see Fig. 1). The string is released and Q begins to descend. When the line OP makes an angle θ radians, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, with the horizontal, the particles have speed $v\,\mathrm{m\,s^{-1}}$ (see Fig. 2).

(i) By considering the total energy of the system, or otherwise, show that

$$v^2 = 6.72\theta - 5.04\sin\theta.$$ [5]

(ii) Show that the magnitude of the contact force between P and the cylinder is

$$(5.46\sin\theta - 3.36\theta) \text{ newtons}.$$

Hence find the value of θ for which the magnitude of the contact force is greatest. [6]

(iii) Find the transverse component of the acceleration of P in terms of θ. [3]

OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT **4730/01**

MATHEMATICS

Mechanics 3

WEDNESDAY 10 JANUARY 2007 Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

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INFORMATION FOR CANDIDATES

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ADVICE TO CANDIDATES

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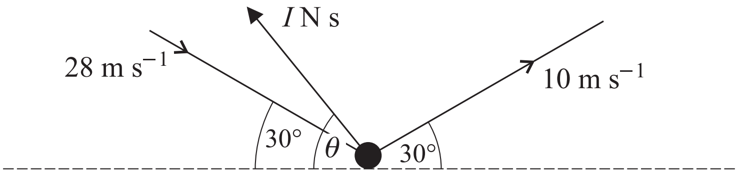
This document consists of **4** printed pages.

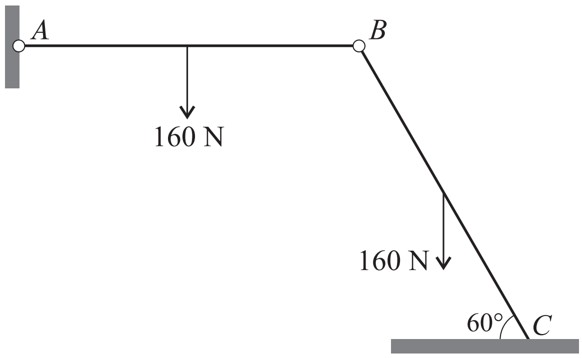
1 A particle P of mass 0.6 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. While hanging at a distance 0.4 m vertically below O, P is projected horizontally with speed $5\,\mathrm{m\,s^{-1}}$ and moves in a complete vertical circle. Calculate the tension in the string when P is vertically above O. [6]

2



When a tennis ball of mass 0.057 kg bounces it receives an impulse of magnitude I N s at an angle of θ to the horizontal. Immediately before the ball bounces it has speed $28\,\mathrm{m\,s^{-1}}$ in a direction of 30° to the horizontal. Immediately after the ball bounces it has speed $10\,\mathrm{m\,s^{-1}}$ in a direction of 30° to the horizontal (see diagram). Find I and θ. [7]

3



Two identical uniform rods, AB and BC, are freely jointed to each other at B, and A is freely jointed to a fixed point. The rods are in limiting equilibrium in a vertical plane, with C resting on a rough horizontal surface. AB is horizontal, and BC is inclined at 60° to the horizontal. The weight of each rod is 160 N (see diagram).

(i) By taking moments for AB about A, find the vertical component of the force on AB at B. Hence or otherwise find the magnitude of the vertical component of the contact force on BC at C. [3]

(ii) Calculate the magnitude of the frictional force on BC at C and state its direction. [4]

(iii) Calculate the value of the coefficient of friction at C. [2]

4 A particle P of mass 0.2 kg is suspended from a fixed point O by a light elastic string of natural length 0.7 m and modulus of elasticity 3.5 N. P is at the equilibrium position when it is projected vertically downwards with speed $1.6\,\mathrm{m\,s^{-1}}$. At time t s after being set in motion P is x m below the equilibrium position and has velocity $v\,\mathrm{m\,s^{-1}}$.

(i) Show that the equilibrium position of P is 1.092 m below O. [3]

(ii) Prove that P moves with simple harmonic motion, and calculate the amplitude. [5]

(iii) Calculate x and v when $t = 0.4$. [5]

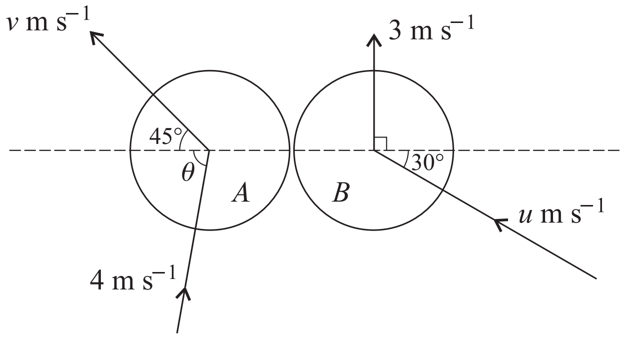
5 The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass m kg and is released from rest. When a grain has fallen a distance x m, it has speed $v\,\mathrm{m\,s^{-1}}$. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude mkv^2 N, where k is a positive constant.

(i) Show that $\left(\dfrac{v}{g - kv^2}\right)\dfrac{dv}{dx} = 1$. [2]

(ii) Find v^2 in terms of k, g and x. Hence show that, as x becomes large, the limiting value of v is $\sqrt{\dfrac{g}{k}}$. [7]

(iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find k. [3]

6



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.4 kg, and B has mass m kg. Immediately before the collision, A is moving with speed $4\,\mathrm{m\,s^{-1}}$ at an acute angle θ to the line of centres, and B is moving with speed $u\,\mathrm{m\,s^{-1}}$ at 30° to the line of centres. Immediately after the collision A is moving with speed $v\,\mathrm{m\,s^{-1}}$ at 45° to the line of centres, and B is moving with speed $3\,\mathrm{m\,s^{-1}}$ perpendicular to the line of centres (see diagram).

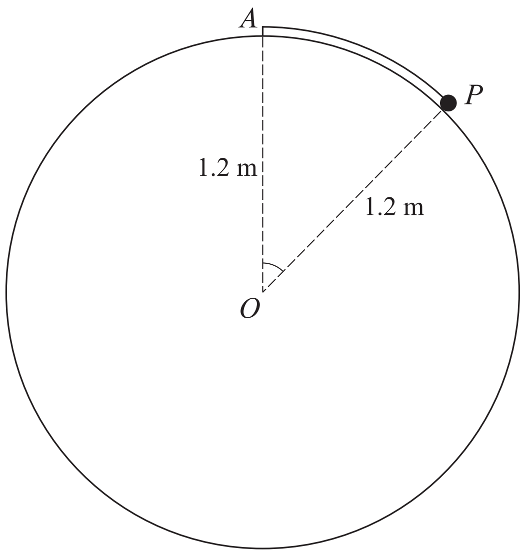
(i) Find u. [2]

(ii) Given that $\theta = 88.1°$ correct to 1 decimal place, calculate the approximate values of v and m. [5]

(iii) The coefficient of restitution is 0.75. Show that the exact value of θ is a root of the equation $8\sin\theta - 6\cos\theta = 9\cos 30°$. [5]

[Question 7 is printed overleaf.]

7



The diagram shows a particle P of mass 0.5 kg attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 1.2 m. The string has natural length 0.6 m and modulus of elasticity 6.86 N. P is released from rest at a point on the surface of the sphere where the acute angle AOP is at least 0.5 radians.

(i) (a) For the case angle $AOP = \alpha$, P remains at rest. Show that $\sin \alpha = 2.8\alpha - 1.4$. [4]

(b) Use the iterative formula

$$\alpha_{n+1} = \frac{\sin \alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find α correct to 2 significant figures. [2]

(ii) Given instead that angle $AOP = 0.5$ radians when P is released, find the speed of P when angle $AOP = 0.8$ radians, given that P is at all times in contact with the surface of the sphere. State whether the speed of P is increasing or decreasing when angle $AOP = 0.8$ radians. [7]

ADVANCED GCE UNIT **4730/01**

MATHEMATICS

Mechanics 3

MONDAY 21 MAY 2007 Morning

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

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ADVICE TO CANDIDATES

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1 A particle P is moving with simple harmonic motion in a straight line. The period is 6.1 s and the amplitude is 3 m. Calculate, in either order,

(i) the maximum speed of P, [3]

(ii) the distance of P from the centre of motion when P has speed $2.5\,\mathrm{m\,s^{-1}}$. [3]

2 A tennis ball of mass 0.057 kg has speed $10\,\mathrm{m\,s^{-1}}$. The ball receives an impulse of magnitude 0.6 N s which reduces the speed of the ball to $7\,\mathrm{m\,s^{-1}}$. Using an impulse-momentum triangle, or otherwise, find the angle the impulse makes with the original direction of motion of the ball. [7]

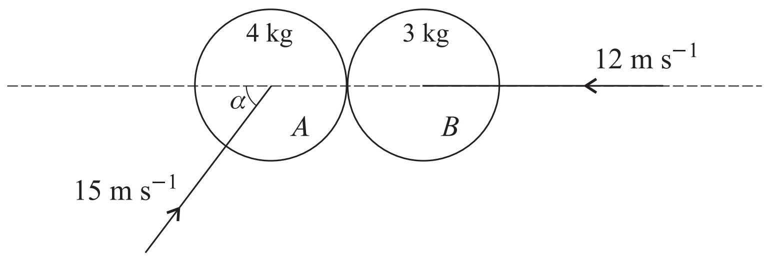
3 A particle P of mass 0.2 kg is projected horizontally with speed $u\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. P moves in a straight line and, at time t s after projection, P has speed $v\,\mathrm{m\,s^{-1}}$ and is x m from O. The only force acting on P has magnitude $0.4v^2$ N and is directed towards O.

(i) Show that $\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}x} = -2$. [2]

(ii) Hence show that $v = u\mathrm{e}^{-2x}$. [4]

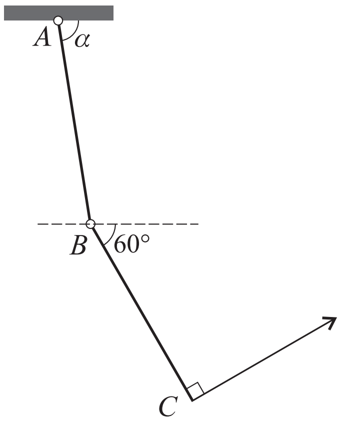
(iii) Find u, given that $x = 2$ when $t = 4$. [4]

4



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 3 kg respectively. They are moving on a horizontal surface, and they collide. Immediately before the collision, A is moving with speed $15\,\mathrm{m\,s^{-1}}$ at an angle α to the line of centres, where $\sin\alpha = 0.8$, and B is moving along the line of centres with speed $12\,\mathrm{m\,s^{-1}}$ (see diagram). The coefficient of restitution between the spheres is 0.5. Find the speed and direction of motion of each sphere after the collision. [10]

5

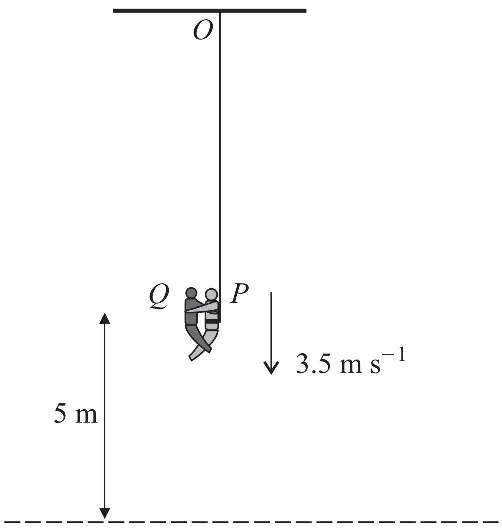


Two uniform rods AB and BC, each of length 1.4 m and weight 80 N, are freely jointed to each other at B, and AB is freely jointed to a fixed point at A. They are held in equilibrium with AB at an angle α to the horizontal, and BC at an angle of $60°$ to the horizontal, by a light string, perpendicular to BC, attached to C (see diagram).

(i) By taking moments about B for BC, calculate the tension in the string. Hence find the horizontal and vertical components of the force acting on BC at B. [7]

(ii) Find α. [4]

6



A circus performer P of mass 80 kg is suspended from a fixed point O by an elastic rope of natural length 5.25 m and modulus of elasticity 2058 N. P is in equilibrium at a point 5 m above a safety net. A second performer Q, also of mass 80 kg, falls freely under gravity from a point above P. P catches Q and together they begin to descend vertically with initial speed 3.5 m s^{-1} (see diagram). The performers are modelled as particles.

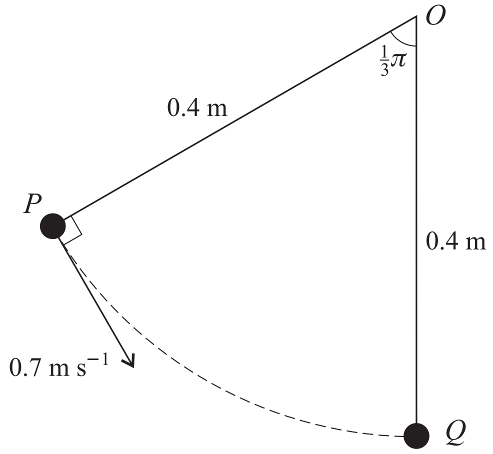
(i) Show that, when P is in equilibrium, $OP = 7.25$ m. [3]

(ii) Verify that P and Q together just reach the safety net. [5]

(iii) At the lowest point of their motion P releases Q. Prove that P subsequently just reaches O. [3]

(iv) State two additional modelling assumptions made when answering this question. [2]

7



A particle P of mass 0.8 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. A particle Q is suspended from O by an identical string. With the string OP taut and inclined at $\frac{1}{3}\pi$ radians to the vertical, P is projected with speed $0.7\,\mathrm{m\,s^{-1}}$ in a direction perpendicular to the string so as to strike Q directly (see diagram). The coefficient of restitution between P and Q is $\frac{1}{7}$.

(i) Calculate the tension in the string immediately after P is set in motion. [4]

(ii) Immediately after P and Q collide they have equal speeds and are moving in opposite directions. Show that Q starts to move with speed $0.15\,\mathrm{m\,s^{-1}}$. [4]

(iii) Prove that before the second collision between P and Q, Q is moving with approximate simple harmonic motion. [5]

(iv) Hence find the time interval between the first and second collisions of P and Q. [2]

ADVANCED GCE **4730/01**

MATHEMATICS

Mechanics 3

THURSDAY 17 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

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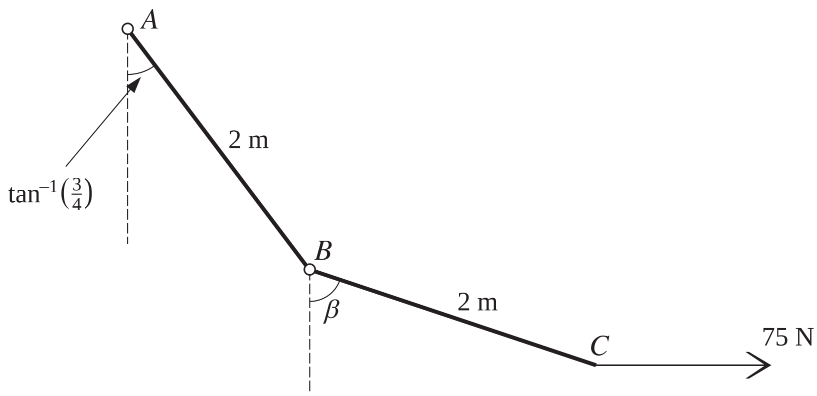
1 A smooth horizontal surface lies in the x-y plane. A particle P of mass 0.5 kg is moving on the surface with speed $5\,\text{m}\,\text{s}^{-1}$ in the x-direction when it is struck by a horizontal blow whose impulse has components -3.5 N s and 2.4 N s in the x-direction and y-direction respectively.

(i) Find the components in the x-direction and the y-direction of the velocity of P immediately after the blow. Hence show that the speed of P immediately after the blow is $5.2\,\text{m}\,\text{s}^{-1}$. [4]

P is struck by a second horizontal blow whose impulse is $\mathbf{I}$.

(ii) Given that P's direction of motion immediately after this blow is parallel to the x-axis, write down the component of $\mathbf{I}$ in the y-direction. [2]

2



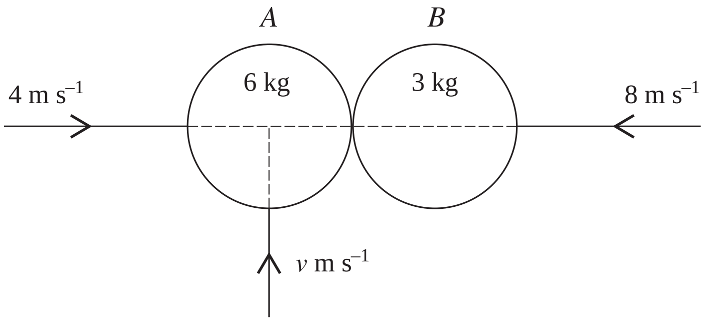
Two uniform rods AB and BC, each of length 2 m, are freely jointed at B. The weights of the rods are W N and 50 N respectively. The end A of AB is hinged at a fixed point. The rods AB and BC make angles $\tan^{-1}\left(\frac{3}{4}\right)$ and β respectively with the downward vertical, and are held in equilibrium in a vertical plane by a horizontal force of magnitude 75 N acting at C (see diagram).

(i) By taking moments about B for BC, show that $\tan\beta = 3$. [3]

(ii) Write down the horizontal and vertical components of the force acting on AB at B. [2]

(iii) Find the value of W. [4]

3



Two uniform smooth spheres A and B, of equal radius, have masses 6 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision the velocity of A has components $4\,\text{m s}^{-1}$ along the line of centres towards B, and $v\,\text{m s}^{-1}$ perpendicular to the line of centres. B is moving with speed $8\,\text{m s}^{-1}$ along the line of centres towards A (see diagram). The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the component of the velocity of A along the line of centres immediately after the collision. [5]

(ii) Given that the speeds of A and B are the same immediately after the collision, and that $3e^2 = 1$, find v. [4]

4 A particle of mass m kg is released from rest at a fixed point O and falls vertically. The particle is subject to an upward resisting force of magnitude $0.49mv$ N where $v\,\text{m s}^{-1}$ is the velocity of the particle when it has fallen a distance of x m from O.

(i) Write down a differential equation for the motion of the particle, and show that the equation can be written as $\left(\frac{20}{20-v} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}x} = 0.49$. [5]

(ii) Hence find an expression for x in terms of v. [5]

5 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $0.75mg$ N. The other end of the string is attached to a fixed point O of a smooth plane inclined at 30° to the horizontal. P is released from rest at O and moves down the plane.

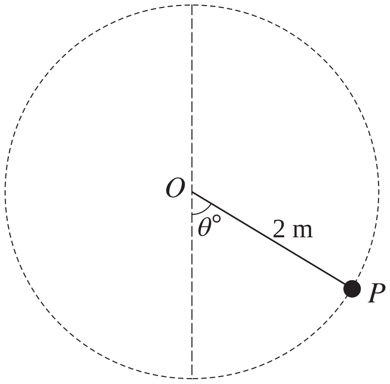
(i) Show that the maximum speed of P is reached when the extension of the string is 0.8 m. [3]

(ii) Find the maximum speed of P. [4]

(iii) Find the maximum displacement of P from O. [4]

[Questions 6 and 7 are printed overleaf.]

6



A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point O. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of P is $7\,\text{m}\,\text{s}^{-1}$.

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ. [8]

7 A particle P, of mass m kg, is attached to one end of a light elastic string of natural length 3.2 m and modulus of elasticity $4mg$ N. The other end of the string is attached to a fixed point A. The particle is released from rest at a point 4.8 m vertically below A. At time t s after P's release P is $(4 + x)$ m below A.

(i) Show that $4\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -49x$. [3]

P's motion is simple harmonic.

(ii) Write down the amplitude of P's motion and show that the string becomes slack instantaneously at intervals of approximately 1.8 s. [4]

A particle Q is attached to one end of a light **inextensible** string of length L m. The other end of the string is attached to a fixed point B. The particle is released from rest with the string taut and inclined at a small angle with the downward vertical. At time t s after Q's release BQ makes an angle of θ radians with the downward vertical.

(iii) Show that $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \approx -\dfrac{g}{L}\theta$. [3]

The period of the simple harmonic motion to which Q's motion approximates is the same as the period of P's motion.

(iv) Given that $\theta = 0.08$ when $t = 0$, find the speed of Q when $t = 0.25$. [5]

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4730/01**

MATHEMATICS

Mechanics 3

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.8 m and modulus of elasticity $1.35mg$ N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. P is held at rest at a point on the surface 3 m from O. The particle is then released. Find

(i) the initial acceleration of P, [3]

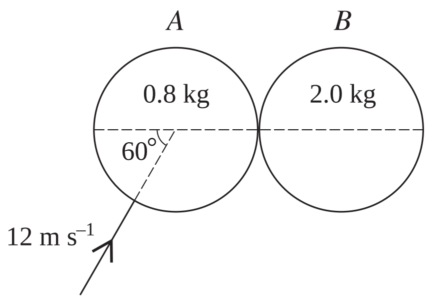
(ii) the speed of P at the instant the string becomes slack. [3]

2 A particle P of mass 0.2 kg is moving with speed $8\,\text{m}\,\text{s}^{-1}$ when it hits a horizontal smooth surface. The direction of motion of P immediately before impact makes an angle of $27°$ with the surface. Given that the coefficient of restitution between the particle and the surface is 0.6, find

(i) the vertical component of the velocity of P immediately after impact, [3]

(ii) the magnitude of the impulse exerted on P. [3]

3



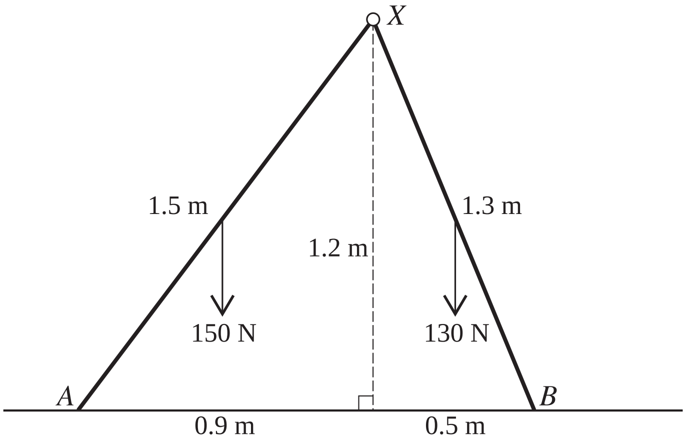
Two uniform smooth spheres A and B, of equal radius, have masses 0.8 kg and 2.0 kg respectively. The spheres are on a horizontal surface. A is moving with speed $12\,\text{m}\,\text{s}^{-1}$ at $60°$ to the line of centres when it collides with B, which is stationary (see diagram). The coefficient of restitution between the spheres is 0.75. Find the speed and direction of motion of A immediately after the collision. [10]

4 A particle P of mass m kg is held at rest at a point O on a fixed plane inclined at an angle $\sin^{-1}\left(\frac{4}{7}\right)$ to the horizontal. P is released and moves down the plane. The total resistance acting on P is $0.2mv$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the velocity of P at time t s after leaving O.

(i) Show that $5\dfrac{dv}{dt} = 28 - v$ and hence find an expression for v in terms of t. [8]

(ii) Find the acceleration of P when $t = 10$. [2]

5



Two uniform rods XA and XB are freely jointed at X. The lengths of the rods are 1.5 m and 1.3 m respectively, and their weights are 150 N and 130 N respectively. The rods are in equilibrium in a vertical plane with A and B in contact with a rough horizontal surface. A and B are at distances horizontally from X of 0.9 m and 0.5 m respectively, and X is 1.2 m above the surface (see diagram).

(i) The normal components of the contact forces acting on the rods at A and B are R_A N and R_B N respectively. Show that $R_A = 125$ and find R_B. [4]

(ii) Find the frictional components of the contact forces acting on the rods at A and B. [4]

(iii) Find the horizontal and vertical components of the force exerted on XA at X, stating their directions. [3]

6 A particle P of mass 0.1 kg moves in a straight line on a smooth horizontal surface. A force of $(0.36 - 0.144x)$ N acts on P in the direction from O to P, where x m is the displacement of P from a point O on the surface at time t s.

(i) By using the substitution $x = y + 2.5$, or otherwise, show that P moves with simple harmonic motion of period 5.24 s, correct to 3 significant figures. [5]

The maximum value of x during the motion is 3.

(ii) Write down the amplitude of P's motion and find the two possible values of x for which P's speed is $0.48\,\mathrm{m\,s^{-1}}$. [4]

(iii) On each of the first two occasions when P has speed $0.48\,\mathrm{m\,s^{-1}}$, P is moving towards O. Find the time interval between

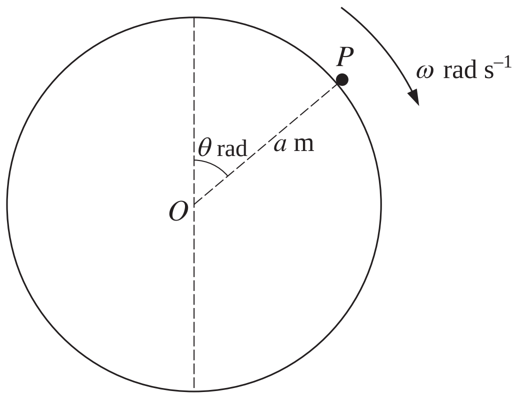
(a) these first two occasions,

(b) the second and third occasions when P has speed $0.48\,\mathrm{m\,s^{-1}}$.

[5]

[Question 7 is printed overleaf.]

7



A particle P of mass m kg is slightly disturbed from rest at the highest point on the surface of a smooth fixed sphere of radius a m and centre O. The particle starts to move downwards on the surface. While P remains on the surface OP makes an angle of θ radians with the upward vertical and has angular speed $\omega\,\text{rad s}^{-1}$ (see diagram). The sphere exerts a force of magnitude R N on P.

(i) Show that $a\omega^2 = 2g(1 - \cos\theta)$. [3]

(ii) Find an expression for R in terms of m, g and θ. [4]

At the instant that P loses contact with the surface of the sphere, find

(iii) the transverse component of the acceleration of P, [4]

(iv) the rate of change of R with respect to time t, in terms of m, g and a. [4]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 19 January 2009
Afternoon

Duration: 1 hour 30 minutes

* 4 7 3 0 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
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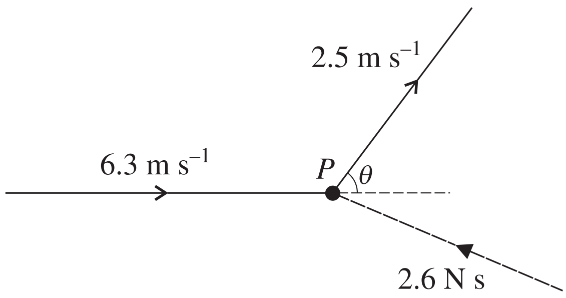
1



A particle P of mass 0.5 kg is moving in a straight line with speed 6.3 m s^{-1}. An impulse of magnitude 2.6 N s applied to P deflects its direction of motion through an angle θ, and reduces its speed to 2.5 m s^{-1} (see diagram). By considering an impulse-momentum triangle, or otherwise,

(i) show that $\cos\theta = 0.6$, **[4]**

(ii) find the angle that the impulse makes with the original direction of motion of P. **[4]**

2

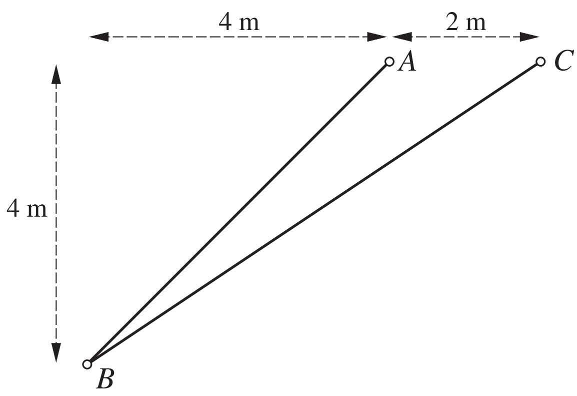


Fig. 1

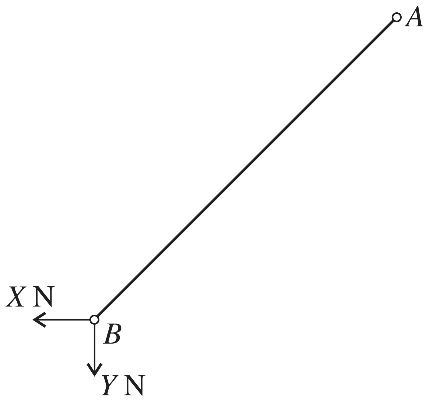


Fig. 2

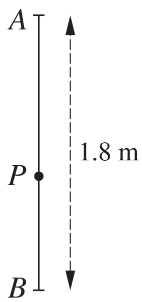
Two uniform rods AB and BC, of weights 70 N and 110 N respectively, are freely jointed at B. The rods are in equilibrium in a vertical plane with A and C at the same horizontal level and $AC = 2$ m. The rod AB is freely jointed to a fixed point at A and the rod BC is freely jointed to a fixed point at C. The horizontal distance between B and A is 4 m and B is 4 m below AC; angle BAC is obtuse (see Fig. 1). The force exerted on the rod AB at B, by the rod BC, has horizontal and vertical components as shown in Fig. 2.

(i) By taking moments about A for the rod AB find the value of $X - Y$. **[2]**

(ii) By taking moments about C for the rod BC show that $2X - 3Y + 165 = 0$. **[2]**

(iii) Find the magnitude of the force acting between AB and BC at B. **[4]**

3



A and B are fixed points with B at a distance of 1.8 m vertically below A. One end of a light elastic string of natural length 0.6 m and modulus of elasticity 24 N is attached to A, and one end of an identical elastic string is attached to B. A particle P of weight 12 N is attached to the other ends of the strings (see diagram).

(i) Verify that P is in equilibrium when it is at a distance of 1.05 m vertically below A. **[2]**

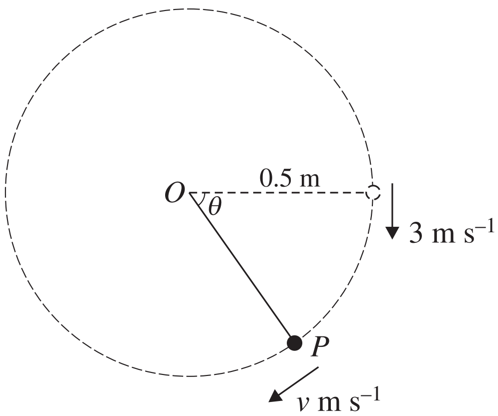
P is released from rest at the point 1.2 m vertically below A and begins to move.

(ii) Show that, when P is x m below its equilibrium position, the tensions in PA and PB are $(18 + 40x)$ N and $(6 - 40x)$ N respectively. **[2]**

(iii) Show that P moves with simple harmonic motion of period 0.777 s, correct to 3 significant figures. **[3]**

(iv) Find the speed with which P passes through the equilibrium position. **[2]**

4



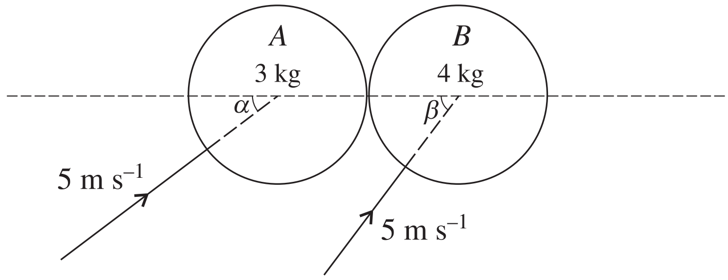
One end of a light inextensible string of length 0.5 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. With the string taut and horizontal, P is projected with a velocity of $3\,\text{m}\,\text{s}^{-1}$ vertically downward. P begins to move in a vertical circle with centre O. While the string remains taut the angular displacement of OP is θ radians from its initial position, and the speed of P is $v\,\text{m}\,\text{s}^{-1}$ (see diagram).

(i) Show that $v^2 = 9 + 9.8\sin\theta$. **[3]**

(ii) Find, in terms of θ, the radial and tangential components of the acceleration of P. **[3]**

(iii) Show that the tension in the string is $(3.6 + 5.88\sin\theta)$ N and hence find the value of θ at the instant when the string becomes slack, giving your answer correct to 1 decimal place. **[4]**

5



Two smooth uniform spheres A and B, of equal radius, have masses 3 kg and 4 kg respectively. They are moving on a horizontal surface, each with speed 5 m s^{-1}, when they collide. The directions of motion of A and B make angles α and β respectively with the line of centres of the spheres, where $\sin\alpha = \cos\beta = 0.6$ (see diagram). The coefficient of restitution between the spheres is 0.75. Find the angle that the velocity of A makes, immediately after impact, with the line of centres of the spheres. **[10]**

6 A stone of mass 0.125 kg falls freely under gravity, from rest, until it has travelled a distance of 10 m. The stone then continues to fall in a medium which exerts an upward resisting force of $0.025v$ N, where v m s^{-1} is the speed of the stone t s after the instant that it enters the resisting medium.

(i) Show by integration that $v = 49 - 35e^{-0.2t}$. **[8]**

(ii) Find how far the stone travels during the first 3 seconds in the medium. **[4]**

7 A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to a fixed point O. The particle is held at rest at O and then released. When the extension of the string is x m, the particle is moving with speed v m s^{-1}.

(i) By considering energy show that $v^2 = 39.2 + 19.6x - 12.5x^2$. **[4]**

(ii) Hence find

(a) the maximum extension of the string, **[2]**

(b) the maximum speed of the particle, **[4]**

(c) the maximum magnitude of the acceleration of the particle. **[5]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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INFORMATION FOR CANDIDATES

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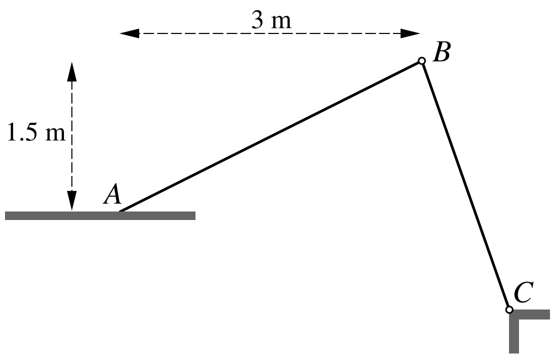
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A smooth sphere of mass 0.3 kg bounces on a fixed horizontal surface. Immediately before the sphere bounces the components of its velocity horizontally and vertically downwards are $4\,\text{m}\,\text{s}^{-1}$ and $6\,\text{m}\,\text{s}^{-1}$ respectively. The speed of the sphere immediately after it bounces is $5\,\text{m}\,\text{s}^{-1}$.

(i) Show that the vertical component of the velocity of the sphere immediately after impact is $3\,\text{m}\,\text{s}^{-1}$, and hence find the coefficient of restitution between the surface and the sphere. **[3]**

(ii) State the direction of the impulse on the sphere and find its magnitude. **[3]**

2

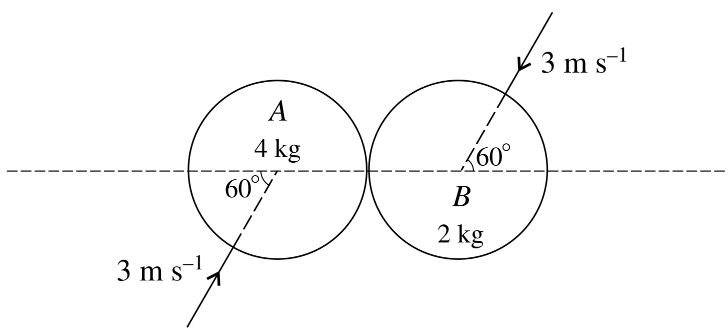


Two uniform rods, AB and BC, are freely jointed to each other at B, and C is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with A resting on a rough horizontal surface. This surface is 1.5 m below the level of B and the horizontal distance between A and B is 3 m (see diagram). The weight of AB is 80 N and the frictional force acting on AB at A is 14 N.

(i) Write down the horizontal component of the force acting on AB at B and show that the vertical component of this force is 33 N upwards. **[4]**

(ii) Given that the force acting on BC at C has magnitude 50 N, find the weight of BC. **[4]**

3



Two uniform smooth spheres A and B, of equal radius, have masses 4 kg and 2 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision both spheres have speed $3\,\text{m}\,\text{s}^{-1}$. The spheres are moving in opposite directions, each at 60° to the line of centres (see diagram). After the collision A moves in a direction perpendicular to the line of centres.

(i) Show that the speed of B is unchanged as a result of the collision, and find the angle that the new direction of motion of B makes with the line of centres. **[8]**

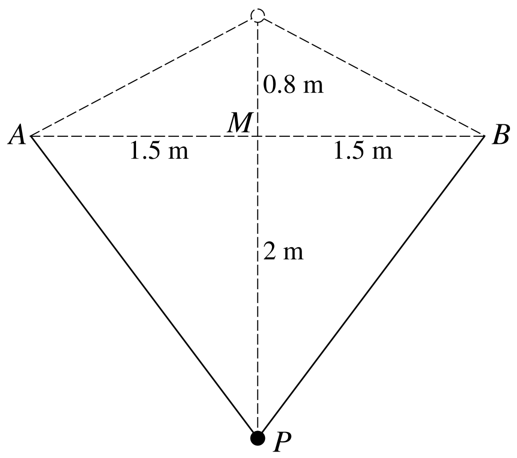
(ii) Find the coefficient of restitution between the spheres. **[2]**

4 A motor-cycle, whose mass including the rider is 120 kg, is decelerating on a horizontal straight road. The motor-cycle passes a point A with speed $40\,\mathrm{m\,s^{-1}}$ and when it has travelled a distance of x m beyond A its speed is $v\,\mathrm{m\,s^{-1}}$. The engine develops a constant power of 8 kW and resistances are modelled by a force of $0.25v^2$ N opposing the motion.

(i) Show that $\dfrac{480v^2}{v^3 - 32\,000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[5]**

(ii) Find the speed of the motor-cycle when it has travelled 500 m beyond A. **[6]**

5



Each of two identical strings has natural length 1.5 m and modulus of elasticity 18 N. One end of one of the strings is attached to A and one end of the other string is attached to B, where A and B are fixed points which are 3 m apart and at the same horizontal level. M is the mid-point of AB. A particle P of mass m kg is attached to the other end of each of the strings. P is held at rest at the point 0.8 m vertically above M, and then released. The lowest point reached by P in the subsequent motion is 2 m below M (see diagram).

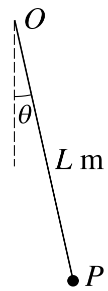
(i) Find the maximum tension in each of the strings during P's motion. **[3]**

(ii) By considering energy,

(a) show that the value of m is 0.42, correct to 2 significant figures, **[5]**

(b) find the speed of P at M. **[3]**

6



A particle P of mass m kg is attached to one end of a light inextensible string of length L m. The other end of the string is attached to a fixed point O. The particle is held at rest with the string taut and then released. P starts to move and in the subsequent motion the angular displacement of OP, at time t s, is θ radians from the downward vertical (see diagram). The initial value of θ is 0.05.

(i) Show that $\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$. **[2]**

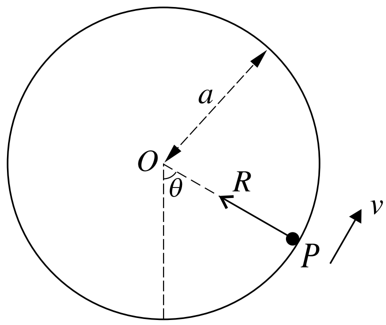
(ii) Hence show that the motion of P is approximately simple harmonic. **[2]**

(iii) Given that the period of the approximate simple harmonic motion is $\frac{4}{7}\pi$ s, find the value of L. **[2]**

(iv) Find the value of θ when $t = 0.7$ s, and the value of t when θ next takes this value. **[4]**

(v) Find the speed of P when $t = 0.7$ s. **[3]**

7



A hollow cylinder has internal radius a. The cylinder is fixed with its axis horizontal. A particle P of mass m is at rest in contact with the smooth inner surface of the cylinder. P is given a horizontal velocity u, in a vertical plane perpendicular to the axis of the cylinder, and begins to move in a vertical circle. While P remains in contact with the surface, OP makes an angle θ with the downward vertical, where O is the centre of the circle. The speed of P is v and the magnitude of the force exerted on P by the surface is R (see diagram).

(i) Find v^2 in terms of u, a, g and θ and show that $R = \frac{mu^2}{a} + mg(3\cos\theta - 2)$. **[7]**

(ii) Given that P just reaches the highest point of the circle, find u^2 in terms of a and g, and show that in this case the least value of v^2 is ag. **[4]**

(iii) Given instead that P oscillates between $\theta = \pm\frac{1}{6}\pi$ radians, find u^2 in terms of a and g. **[2]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

INSTRUCTIONS TO CANDIDATES

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INFORMATION FOR CANDIDATES

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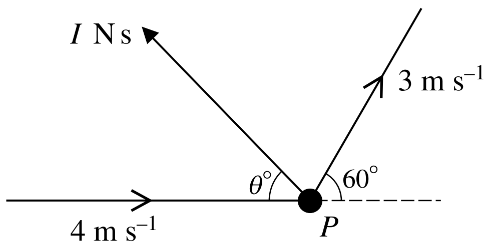
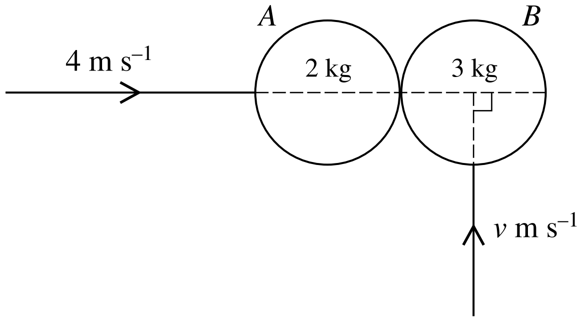
- This document consists of **4** pages. Any blank pages are indicated.

1



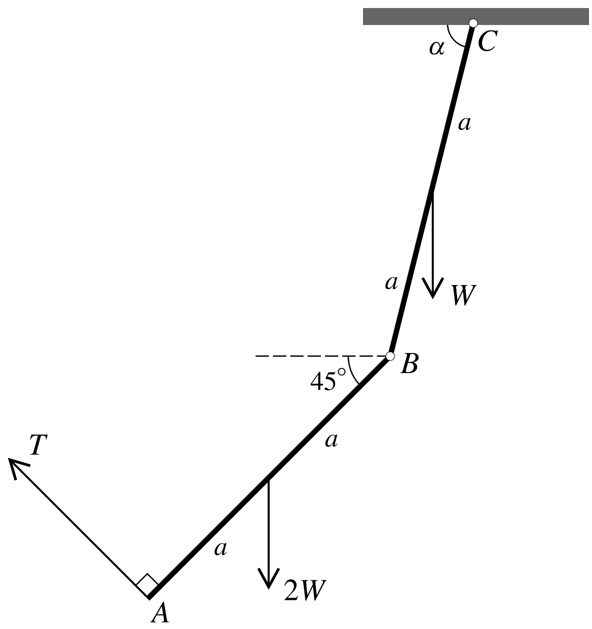
A particle P of mass 0.4 kg is moving horizontally with speed 4 m s^{-1} when it receives an impulse of magnitude I N s, in a direction which makes an angle $(180 - \theta)°$ with the direction of motion of P. Immediately after the impulse acts P moves horizontally with speed 3 m s^{-1}. The direction of motion of P is turned through an angle of 60° by the impulse (see diagram). Find I and θ. **[7]**

2



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed 4 m s^{-1} and is moving along the line of centres, and B has speed v m s^{-1} and is moving perpendicular to the line of centres (see diagram). The coefficient of restitution is 0.6. The direction of motion of B after the collision makes an angle of 45° with the line of centres. Find the value of v. **[7]**

3



Two uniform rods AB and BC, each of length $2a$, have weights $2W$ and W respectively. The rods are freely jointed to each other at B, and BC is freely jointed to a fixed point at C. The rods are held in equilibrium in a vertical plane by a light string attached to A and perpendicular to AB. The rods AB and BC make angles 45° and α, respectively, with the horizontal. The tension in the string is T (see diagram).

(i) By taking moments about B for AB, show that $W = \sqrt{2}T$. **[3]**

(ii) Find the value of $\tan\alpha$. **[6]**

4 A particle P of mass 0.2 kg travels in a straight line on a horizontal surface. It passes through a point O on the surface with speed $2\,\text{m}\,\text{s}^{-1}$. A resistive force of magnitude $0.2(v + v^2)$ N acts on P in the direction opposite to its motion, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of P when it is at a distance x m from O.

(i) Show that $\dfrac{1}{1+v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$. **[3]**

(ii) By solving the differential equation in part **(i)** show that $\dfrac{-\mathrm{e}^x}{3-\mathrm{e}^x}\dfrac{\mathrm{d}x}{\mathrm{d}t} = -1$, where t s is the time taken for P to travel x m from O. **[5]**

(iii) Hence find the value of t when $x = 1$. **[3]**

5 A light elastic string of natural length 1.6 m has modulus of elasticity 120 N. One end of the string is attached to a fixed point O and the other end is attached to a particle P of weight 1.5 N. The particle is released from rest at the point A, which is 2.1 m vertically below O. It comes instantaneously to rest at B, which is vertically above O.

(i) Verify that the distance AB is 4 m. **[4]**

(ii) Find the maximum speed of P during its upward motion from A to B. **[7]**

6

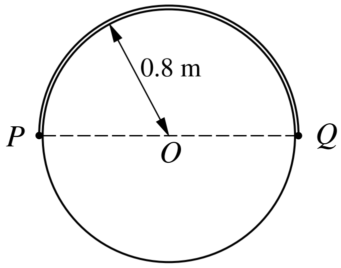


Fig. 1

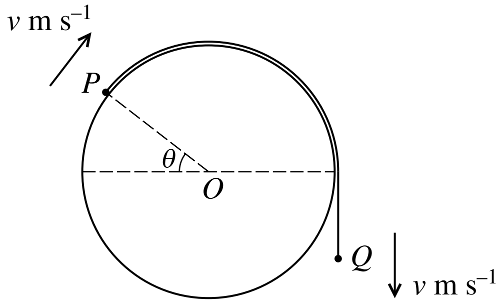


Fig. 2

A light inextensible string of length 0.8π m has particles P and Q, of masses 0.4 kg and 0.58 kg respectively, attached to its ends. The string passes over a smooth horizontal cylinder of radius 0.8 m, which is fixed with its axis horizontal and passing through a fixed point O. The string is held at rest in a vertical plane perpendicular to the axis of the cylinder, with P and Q at opposite ends of the horizontal diameter of the cylinder through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is v m s^{-1} (see Fig. 2).

(i) By considering the total energy of the system, obtain an expression for v^2 in terms of θ. **[5]**

(ii) Show that the magnitude of the force exerted on P by the cylinder is $(7.12 \sin\theta - 4.64\theta)$ N. **[4]**

(iii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $1.53 < \alpha < 1.54$. **[4]**

7 A particle P of mass 0.5 kg is attached to one end of each of two identical light elastic strings of natural length 1.6 m and modulus of elasticity 19.6 N. The other ends of the strings are attached to fixed points A and B on a line of greatest slope of a smooth plane inclined at 30° to the horizontal. The distance AB is 4.8 m and A is higher than B.

(i) Find the distance AP for which P is in equilibrium on the line AB. **[5]**

P is released from rest at a point on AB where both strings are taut. The strings remain taut during the subsequent motion of P and t seconds after release the distance AP is $(2.5 + x)$ m.

(ii) Use Newton's second law to obtain an equation of the form $\frac{d^2x}{dt^2} = kx$. State the property of the constant k for which the equation indicates that P's motion is simple harmonic, and find the period of this motion. **[5]**

(iii) Given that $x = 0.5$ when $t = 0$, find the values of x for which the speed of P is 2.8 m s^{-1}. **[4]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS 4730

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
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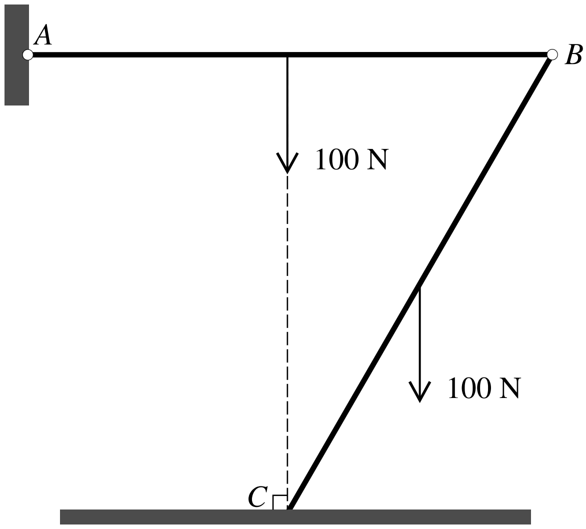
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 A small ball of mass 0.8 kg is moving with speed $10.5\,\mathrm{m\,s^{-1}}$ when it receives an impulse of magnitude 4 N s. The speed of the ball immediately afterwards is $8.5\,\mathrm{m\,s^{-1}}$. The angle between the directions of motion before and after the impulse acts is α. Using an impulse-momentum triangle, or otherwise, find α. **[6]**

2



Two uniform rods AB and BC are of equal length and each has weight 100 N. The rods are freely jointed to each other at B, and A is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with AB horizontal and C resting on a rough horizontal surface. C is vertically below the mid-point of AB (see diagram).

(i) By taking moments about A for AB, find the vertical component of the force on AB at B. Hence find the vertical component of the contact force on BC at C. **[3]**

(ii) Calculate the magnitude of the frictional force on BC at C and state its direction. **[4]**

3

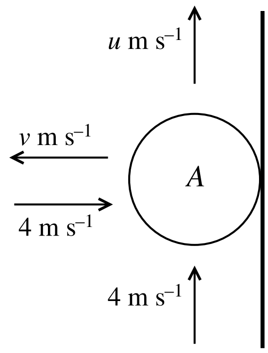


Fig. 1

A uniform smooth sphere A moves on a smooth horizontal surface towards a smooth vertical wall. Immediately before the sphere hits the wall it has components of velocity parallel and perpendicular to the wall each of magnitude $4\,\mathrm{m\,s^{-1}}$. Immediately after hitting the wall the components have magnitudes $u\,\mathrm{m\,s^{-1}}$ and $v\,\mathrm{m\,s^{-1}}$, respectively (see Fig. 1).

(i) Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, state the values of u and v. **[2]**

Shortly after hitting the wall the sphere A comes into contact with another uniform smooth sphere B, which has the same mass and radius as A. The sphere B is stationary and at the instant of contact the line of centres of the spheres is parallel to the wall (see Fig. 2). The contact between the spheres is perfectly elastic.

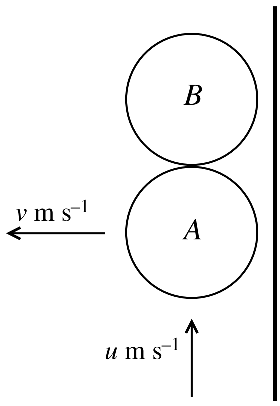


Fig. 2

(ii) Find, for each sphere, its speed and its direction of motion immediately after the contact. **[6]**

4 O is a fixed point on a horizontal plane. A particle P of mass 0.25 kg is released from rest at O and moves in a straight line on the plane. At time t s after release the only horizontal force acting on P has magnitude

$$\tfrac{1}{2400}(144 - t^2)\text{ N} \quad \text{for } 0 \leqslant t \leqslant 12$$

and

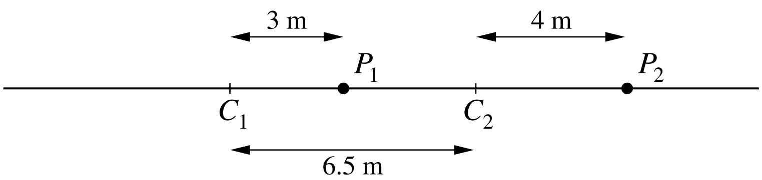
$$\tfrac{1}{2400}(t^2 - 144)\text{ N} \quad \text{for } t \geqslant 12.$$

The force acts in the direction of P's motion. P's velocity at time t s is $v\,\text{m s}^{-1}$.

(i) Find an expression for v in terms of t, valid for $t \geqslant 12$, and hence show that v is three times greater when $t = 24$ than it is when $t = 12$. **[8]**

(ii) Sketch the (t, v) graph for $0 \leqslant t \leqslant 24$. **[3]**

5



Particles P_1 and P_2 are each moving with simple harmonic motion along the same straight line. P_1's motion has centre C_1, period 2π s and amplitude 3 m; P_2's motion has centre C_2, period $\frac{4}{3}\pi$ s and amplitude 4 m. The points C_1 and C_2 are 6.5 m apart. The displacements of P_1 and P_2 from their centres of oscillation at time t s are denoted by x_1 m and x_2 m respectively. The diagram shows the positions of the particles at time $t = 0$, when $x_1 = 3$ and $x_2 = 4$.

(i) State expressions for x_1 and x_2 in terms of t, which are valid until the particles collide. **[3]**

The particles collide when $t = 5.99$, correct to 3 significant figures.

(ii) Find the distance travelled by P_2 before the collision takes place. **[4]**

(iii) Find the velocities of P_1 and P_2 immediately before the collision, and state whether the particles are travelling in the same direction or in opposite directions. **[4]**

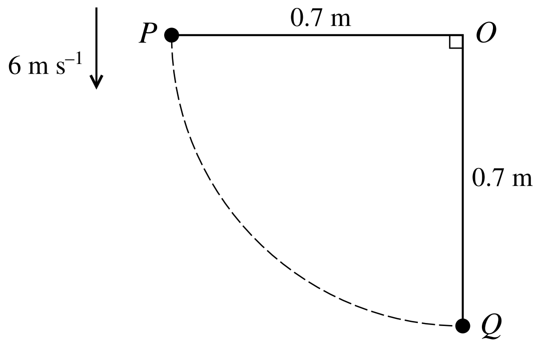
6 A bungee jumper of weight W N is joined to a fixed point O by a light elastic rope of natural length 20 m and modulus of elasticity 32 000 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Given that the jumper just reaches a point 25 m below O, find the value of W. **[5]**

(ii) Find the maximum speed reached by the jumper. **[4]**

(iii) Find the maximum value of the deceleration of the jumper during the downward motion. **[3]**

7



A particle P is attached to a fixed point O by a light inextensible string of length 0.7 m. A particle Q is in equilibrium suspended from O by an identical string. With the string OP taut and horizontal, P is projected vertically downwards with speed $6\,\mathrm{m\,s^{-1}}$ so that it strikes Q directly (see diagram). P is brought to rest by the collision and Q starts to move with speed $4.9\,\mathrm{m\,s^{-1}}$.

(i) Find the speed of P immediately before the collision. Hence find the coefficient of restitution between P and Q. **[3]**

(ii) Given that the speed of Q is $v\,\mathrm{m\,s^{-1}}$ when OQ makes an angle θ with the downward vertical, find an expression for v^2 in terms of θ, and show that the tension in the string OQ is $14.7m(1 + 2\cos\theta)$ N, where m kg is the mass of Q. **[6]**

(iii) Find the radial and transverse components of the acceleration of Q at the instant that the string OQ becomes slack. **[4]**

(iv) Show that $V^2 = 0.8575$, where $V\,\mathrm{m\,s^{-1}}$ is the speed of Q when it reaches its greatest height (after the string OQ becomes slack). Hence find the greatest height reached by Q above its initial position. **[4]**

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ADVANCED GCE

MATHEMATICS

4730

Mechanics 3

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 24 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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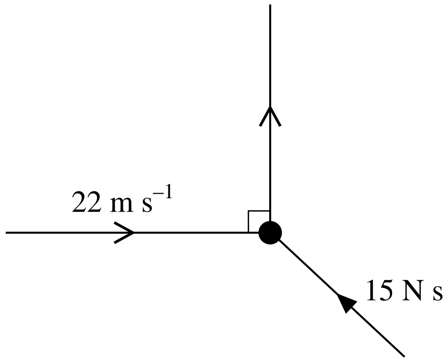
1



A ball of mass 0.5 kg is moving with speed 22 m s^{-1} in a straight line when it is struck by a bat. The impulse exerted by the bat has magnitude 15 N s and the ball is deflected through an angle of 90° (see diagram). Find

(i) the direction of the impulse, **[3]**

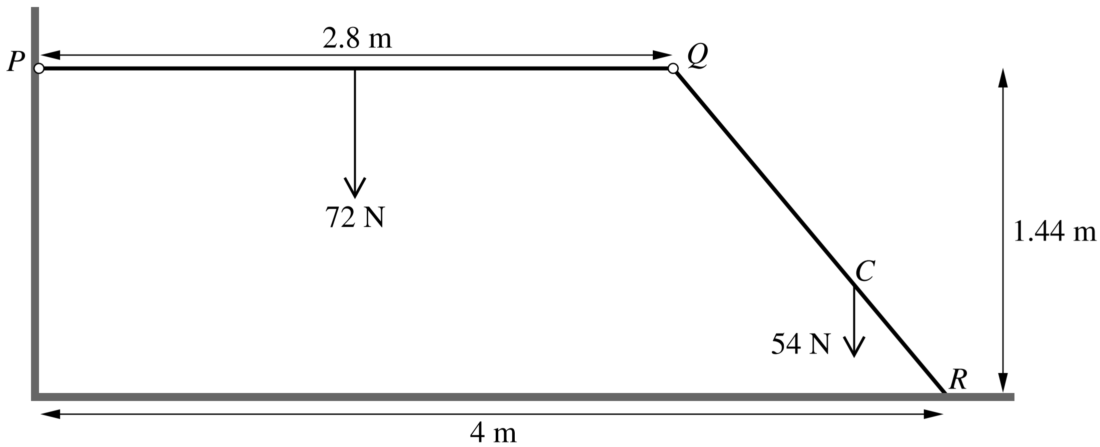
(ii) the speed of the ball immediately after it is struck. **[3]**

2 A particle of mass 0.4 kg is attached to a fixed point O by a light inextensible string of length 0.5 m. The particle is projected horizontally with speed 6 m s^{-1} from the point 0.5 m vertically below O. The particle moves in a complete circle. Find the tension in the string when

(i) the string is horizontal,

(ii) the particle is vertically above O. **[6]**

3



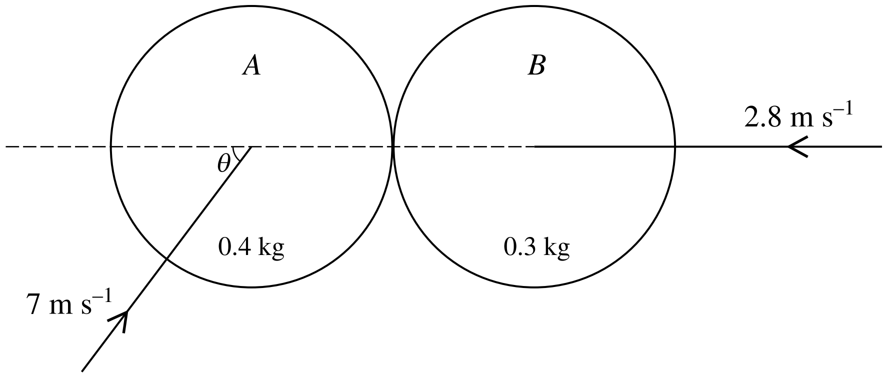
A uniform rod PQ has weight 72 N. A non-uniform rod QR has weight 54 N and its centre of mass is at C, where $QC = 2CR$. The rods are freely jointed to each other at Q. The rod PQ is freely jointed to a fixed point of a vertical wall at P and the rod QR rests on horizontal ground at R. The rod PQ is 2.8 m long and is horizontal. The point R is 1.44 m below the level of PQ and 4 m from the wall (see diagram).

(i) Find the vertical component of the force exerted by the wall on PQ. **[2]**

(ii) Hence show that the normal component of the force exerted by the ground on QR is 90 N. **[2]**

(iii) Given that the friction at R is limiting, find the coefficient of friction between the rod QR and the ground. **[5]**

4



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.4 kg and B has mass 0.3 kg. Immediately before the collision A is moving with speed 7 m s^{-1} at an acute angle θ to the line of centres, where $\cos\theta = 0.6$, and B is moving with speed 2.8 m s^{-1} along the line of centres (see diagram). The coefficient of restitution between the spheres is 0.7. Find

(i) the speed of B immediately after the collision, **[6]**

(ii) the angle turned through by the direction of motion of A as a result of the collision. **[5]**

5 A particle P of mass 0.05 kg is suspended from a fixed point O by a light elastic string of natural length 0.5 m and modulus of elasticity 2.45 N.

(i) Show that the equilibrium position of P is 0.6 m below O. **[3]**

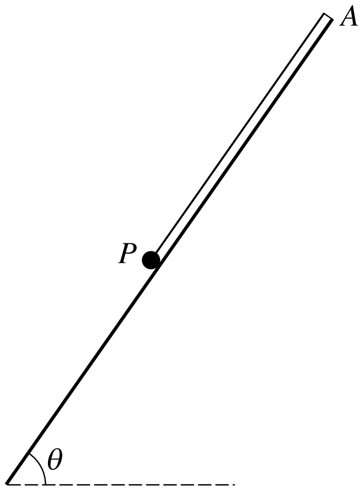
P is held at rest at a point 0.675 m vertically below O and then released. At time t s after P is released, its downward displacement from the equilibrium position is x m.

(ii) Show that $\dfrac{d^2x}{dt^2} = -98x$. **[3]**

(iii) Find the value of x and the magnitude and direction of the velocity of P when $t = 0.2$. **[7]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P, of mass 3.5 kg, is in equilibrium suspended from the top A of a smooth slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{40}{49}$, by an elastic rope of natural length 4 m and modulus of elasticity 112 N (see diagram). Another particle Q, of mass 0.5 kg, is released from rest at A and slides freely downwards until it reaches P and becomes attached to it.

(i) Find the value of V^2, where $V\,\text{m s}^{-1}$ is the speed of Q immediately before it becomes attached to P, and show that the speed of the combined particles, immediately after Q becomes attached to P, is $\frac{1}{2}\sqrt{5}\,\text{m s}^{-1}$. **[6]**

The combined particles slide downwards for a distance of X m, before coming instantaneously to rest at B.

(ii) Show that $28X^2 - 8X - 5 = 0$. **[6]**

7 A particle P of mass 0.2 kg is released from rest at a point O and falls vertically. Air resistance of magnitude $\frac{v^2}{2000}$ N acts upwards on P, where $v\,\text{m s}^{-1}$ is the velocity of P when it has fallen a distance of x m.

(i) Show that $\left(\frac{400v}{3920 - v^2}\right)\frac{dv}{dx} = 1$. **[2]**

(ii) Find v^2 in terms of x and hence show that $v^2 < 3920$ for all values of x. **[7]**

(iii) Find the work done against the air resistance while P is falling, from O, to the point where its downward acceleration is $5.8\,\text{m s}^{-2}$. **[6]**

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Mechanics 3

4730

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Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

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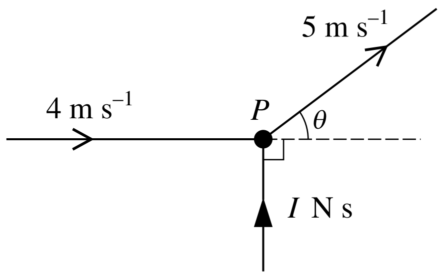
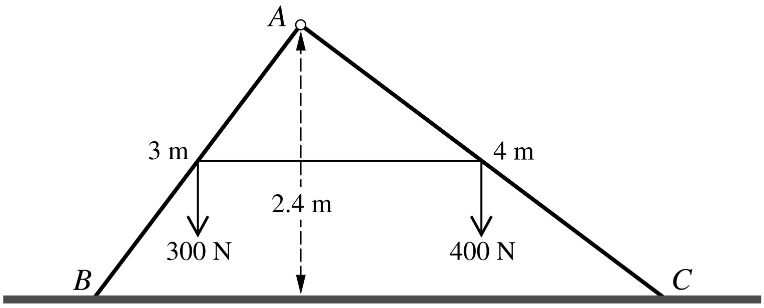
1



A particle P of mass 0.3 kg is moving in a straight line with speed 4 m s^{-1} when it is deflected through an angle θ by an impulse of magnitude I N s. The impulse acts at right angles to the initial direction of motion of P (see diagram). The speed of P immediately after the impulse acts is 5 m s^{-1}. Show that $\cos\theta = 0.8$ and find the value of I. **[4]**

2



Two uniform rods AB and AC, of lengths 3 m and 4 m respectively, have weights 300 N and 400 N respectively. The rods are freely jointed at A. The mid-points of the rods are joined by a light inextensible string. The rods are in equilibrium in a vertical plane with the string taut and B and C in contact with a smooth horizontal surface. The point A is 2.4 m above the surface (see diagram).

(i) Show that the force exerted by the surface on AB is 374 N and find the force exerted by the surface on AC. **[4]**

(ii) Find the tension in the string. **[3]**

(iii) Find the horizontal and vertical components of the force exerted on AB at A and state their directions. **[3]**

3 A particle P of mass 0.25 kg is projected horizontally with speed 5 m s^{-1} from a fixed point O on a smooth horizontal surface and moves in a straight line on the surface. The only horizontal force acting on P has magnitude $0.2v^2$ N, where v m s^{-1} is the velocity of P at time t s after it is projected from O. This force is directed towards O.

(i) Find an expression for v in terms of t. **[5]**

The particle P passes through a point X with speed 0.2 m s^{-1}.

(ii) Find the average speed of P for its motion between O and X. **[5]**

4 One end of a light inextensible string of length 2 m is attached to a fixed point O. A particle P of mass 0.2 kg is attached to the other end of the string. P is held at rest with the string taut so that OP makes an angle of 0.15 radians with the downward vertical. P is released and t seconds afterwards OP makes an angle of θ radians with the downward vertical.

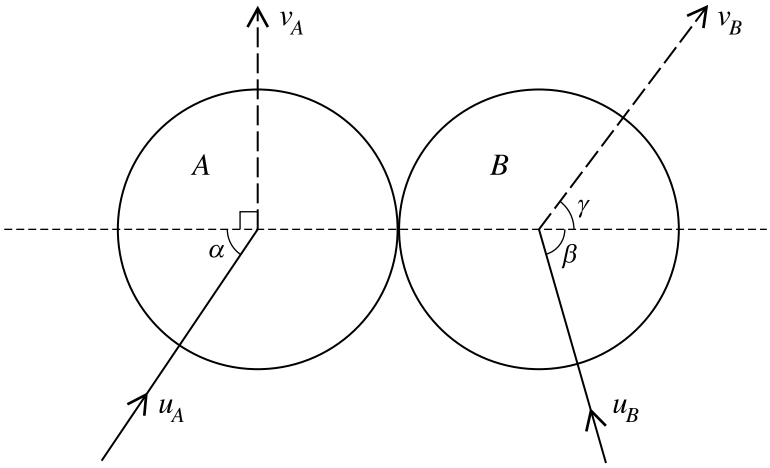
(i) Show that $\frac{d^2\theta}{dt^2} = -4.9 \sin\theta$ and give a reason why the motion is approximately simple harmonic. **[3]**

Using the simple harmonic approximation,

(ii) obtain an expression for θ in terms of t and hence find the values of t at the first and second occasions when $\theta = -0.1$, **[5]**

(iii) find the angular speed of OP and the linear speed of P when $t = 0.5$. **[3]**

5



Two uniform smooth identical spheres A and B are moving towards each other on a horizontal surface when they collide. Immediately before the collision A and B are moving with speeds u_A m s^{-1} and u_B m s^{-1} respectively, at acute angles α and β, respectively, to the line of centres. Immediately after the collision A and B are moving with speeds v_A m s^{-1} and v_B m s^{-1} respectively, at right angles and at acute angle γ, respectively, to the line of centres (see diagram).

(i) Given that $\sin\beta = 0.96$ and $\frac{v_B}{u_B} = 1.2$, find the value of $\sin\gamma$. **[2]**

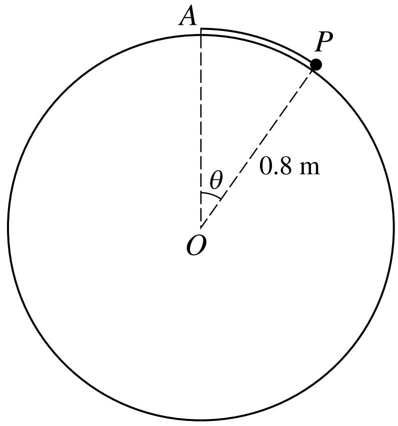
(ii) Given also that, before the collision, the component of A's velocity parallel to the line of centres is 2 m s^{-1}, find the values of u_B and v_B. **[5]**

(iii) Find the coefficient of restitution between the spheres. **[3]**

(iv) Given that the kinetic energy of A immediately before the collision is $6.5m$ J, where m kg is the mass of A, find the value of v_A. **[2]**

[Questions 6 and 7 are printed overleaf.]

6



A particle P of weight 6 N is attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 0.8 m. The string has natural length $\frac{1}{10}\pi$ m and modulus of elasticity 9 N. P is released from rest at a point X on the sphere where OX makes an angle of $\frac{1}{4}\pi$ radians with the upwards vertical. P remains in contact with the sphere as it moves upwards to A. At time t seconds after the release, OP makes an angle of θ radians with the upwards vertical (see diagram). When $\theta = \frac{1}{6}\pi$, P passes through the point Y.

(i) Show that as P moves from X to Y its gravitational potential energy increases by $2.4\left(\sqrt{3} - \sqrt{2}\right)$ J and the elastic potential energy in the string decreases by 0.4π J. **[5]**

(ii) Verify that the transverse acceleration of P is zero when $\theta = \frac{1}{6}\pi$, and hence find the maximum speed of P. **[6]**

7 One end of a light inextensible string of length 0.8 m is attached to a fixed point O. A particle P of mass 0.3 kg is attached to the other end of the string. P is projected horizontally from the point 0.8 m vertically below O with speed 5.6 m s^{-1}. P starts to move in a vertical circle with centre O. The speed of P is v m s^{-1} when the string makes an angle θ with the downward vertical.

(i) While the string remains taut, show that $v^2 = 15.68(1 + \cos\theta)$, and find the tension in the string in terms of θ. **[7]**

(ii) For the instant when the string becomes slack, find the value of θ and the value of v. **[3]**

(iii) Find, in either order, the speed of P when it is at its greatest height after the string becomes slack, and the greatest height reached by P above its point of projection. **[4]**

OCR
RECOGNISING ACHIEVEMENT

Monday 23 January 2012 – Morning

A2 GCE MATHEMATICS

4730 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
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- You are permitted to use a scientific or graphical calculator in this paper.
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- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

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- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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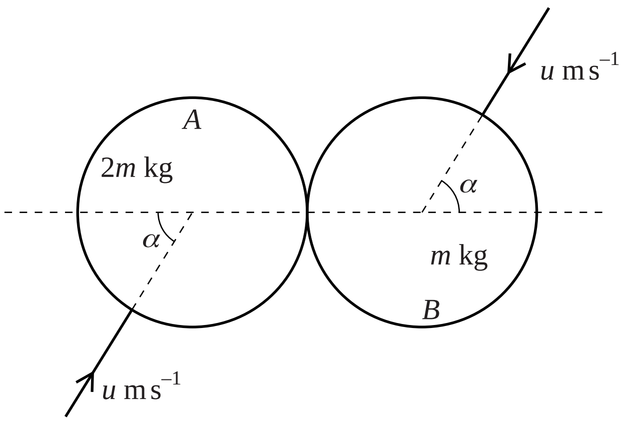
1 A particle P of mass 0.05 kg is moving on a smooth horizontal surface with speed $2\,\mathrm{m\,s^{-1}}$, when it is struck by a horizontal blow in a direction perpendicular to its direction of motion. The magnitude of the impulse of the blow is I N s. The speed of P after the blow is $2.5\,\mathrm{m\,s^{-1}}$.

(i) Find the value of I. **[4]**

Immediately before the blow P is moving parallel to a smooth vertical wall. After the blow P hits the wall and rebounds from the wall with speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$.

(ii) Find the coefficient of restitution between P and the wall. **[4]**

2



Two uniform smooth spheres A and B, of equal radius, have masses $2m$ kg and m kg respectively. They are moving in opposite directions on a horizontal surface and they collide. Immediately before the collision, each sphere has speed $u\,\mathrm{m\,s^{-1}}$ in a direction making an angle α with the line of centres (see diagram). The coefficient of restitution between A and B is 0.5.

(i) Show that the speed of B is unchanged as a result of the collision. **[5]**

(ii) Find the direction of motion of each of the spheres after the collision. **[3]**

3 A particle P of mass 0.3 kg is projected horizontally with speed $u\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving with speed $v\,\mathrm{m\,s^{-1}}$. There is a force of magnitude $1.2v^3$ N resisting the motion of P.

(i) Find an expression for $\dfrac{dv}{dx}$ in terms of v and hence show that $v = \dfrac{u}{4ux + 1}$. **[5]**

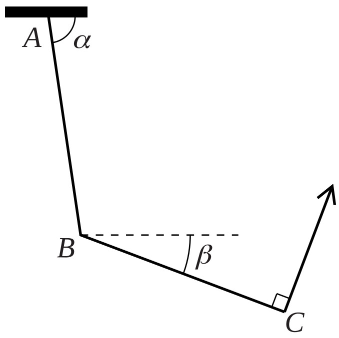
(ii) Given that $x = 2$ when $t = 9$ find the value of u. **[4]**

4 One end of a light elastic string, of natural length 0.75 m and modulus of elasticity 44.1 N, is attached to a fixed point O. A particle P of mass 1.8 kg is attached to the other end of the string. P is released from rest at O and falls vertically. Assuming there is no air resistance, find

(i) the extension of the string when P is at its lowest position, **[4]**

(ii) the acceleration of P at its lowest position. **[4]**

5



Two uniform rods AB and BC, each of length $2L$ m and of weight 84.5 N, are freely jointed at B, and AB is freely jointed to a fixed point at A. The rods are held in equilibrium in a vertical plane by a light string attached at C and perpendicular to BC. The rods AB and BC make angles α and β to the horizontal, respectively (see diagram). It is given that $\cos\beta = \frac{12}{13}$.

(i) Find the tension in the string. **[3]**

(ii) Hence show that the force acting on BC at B has horizontal component of magnitude 15 N and vertical component of magnitude 48.5 N, and state the direction of the component in each case. **[4]**

(iii) Find α. **[4]**

6 A particle P starts from rest at a point A and moves in a straight line with simple harmonic motion. At time t s after the motion starts, P's displacement from a point O on the line is x m towards A. The particle P returns to A for the first time when $t = 0.4\pi$. The maximum speed of P is $4\,\mathrm{m\,s^{-1}}$ and occurs when P passes through O.

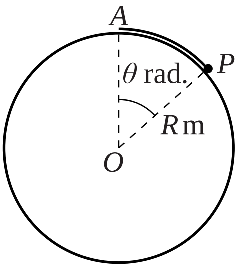
(i) Find the distance OA. **[4]**

(ii) Find the value of x and the velocity of P when $t = 1$. **[4]**

(iii) Find the number of occasions in the interval $0 < t < 1$ at which P's speed is the same as that when $t = 1$, and find the corresponding values of x and t. **[5]**

[Question 7 is printed overleaf.]

7



One end of a light elastic string, of natural length $\frac{2}{3}R$ m and with modulus of elasticity $1.2mg$ N, is attached to the highest point A of a smooth fixed sphere with centre O and radius R m. A particle P of mass m kg is attached to the other end of the string and is in contact with the surface of the sphere, where the angle AOP is equal to θ radians (see diagram).

(i) Given that P is in equilibrium at the point where $\theta = \alpha$, show that $1.8\alpha - \sin\alpha - 1.2 = 0$. Hence show that $\alpha = 1.18$ correct to 3 significant figures. **[7]**

P is now released from rest at the point of the surface of the sphere where $\theta = \frac{2}{3}$, and starts to move downwards on the surface. For an instant when $\theta = \alpha$,

(ii) state the direction of the acceleration of P, **[1]**

(iii) find the magnitude of the acceleration of P. **[7]**

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS

4730 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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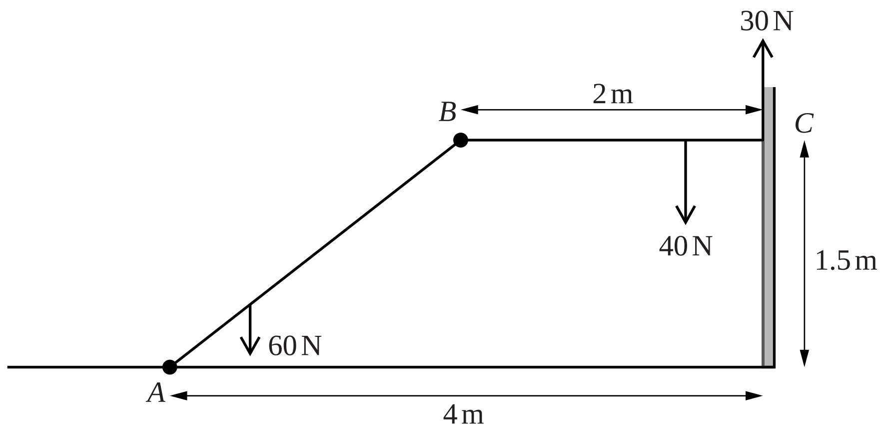
1



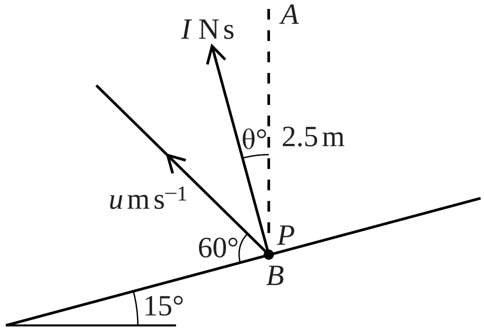
Two non-uniform rods AB and BC have weights 60 N and 40 N respectively. The rods are freely jointed to each other at B. The rod AB is freely jointed to a fixed point on horizontal ground at A and the rod BC rests against a vertical wall at C. The rod BC, whose length is 2 m, is horizontal at a height of 1.5 m above the ground. The point A is 4 m from the wall. The frictional force exerted on BC at C has magnitude 30 N (see diagram). The coefficient of friction between the rod BC and the wall is 0.75.

(i) Find the distance of the centre of mass of BC from B. **[2]**

(ii) Given that the rod BC is on the point of slipping downwards at C, find the magnitude and direction of both the vertical component and the horizontal component of the force exerted on AB at B. **[4]**

(iii) Find the distance of the centre of mass of AB from A. **[3]**

2



B is a point on a smooth plane surface inclined at an angle of 15° to the horizontal. A particle P of mass 0.45 kg is released from rest at the point A which is 2.5 m vertically above B. The particle P rebounds from the surface at an angle of 60° to the line of greatest slope through B, with a speed of u m s^{-1}. The impulse exerted on P by the surface has magnitude I N s and is in a direction making an angle of $\theta°$ with the upward vertical through B (see diagram).

(i) Explain why $\theta = 15$. **[1]**

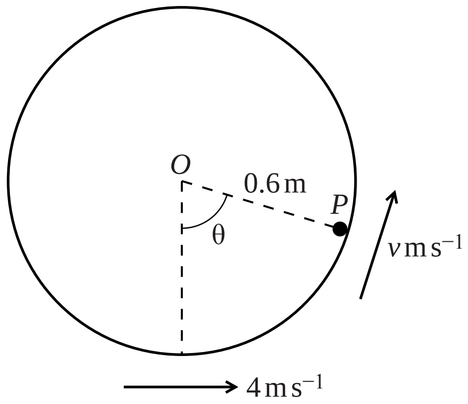
(ii) Find the values of u and I. **[7]**

3 A particle P of mass m kg is released from rest and falls vertically. When P has fallen a distance of x m it has a speed of v m s^{-1}. The only forces acting on P are its weight and air resistance of magnitude $\frac{1}{400}mv^2$ N.

(i) Find v^2 in terms of x and show that v^2 must be less than 3920. **[8]**

(ii) Find the speed of P when it has fallen 100 m. **[2]**

4



A hollow cylinder is fixed with its axis horizontal. The inner surface of the cylinder is smooth and has radius 0.6 m. A particle P of mass 0.45 kg is projected horizontally with speed 4 m s^{-1} from the lowest point of a vertical cross-section of the cylinder and moves in the plane of the cross-section, which is perpendicular to the axis of the cylinder. While P remains in contact with the surface, its speed is v m s^{-1} when OP makes an angle θ with the downward vertical at O, where O is the centre of the cross-section (see diagram). The force exerted on P by the surface is R N.

(i) Show that $v^2 = 4.24 + 11.76\cos\theta$ and find an expression for R in terms of θ. **[6]**

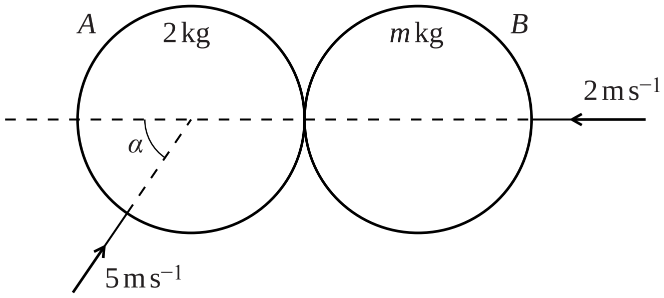
(ii) Find the speed of P at the instant when it leaves the surface. **[4]**

5 One end of a light elastic string, of natural length 0.78 m and modulus of elasticity 0.8 mg N, is attached to a fixed point O on a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{5}{13}$. A particle P of mass m kg is attached to the other end of the string. P is released from rest at O and moves down the plane without reaching the bottom. Find

(i) the maximum speed of P in the subsequent motion, **[6]**

(ii) the distance of P from O when it is at its lowest point. **[4]**

6



Two smooth uniform spheres A and B, of equal radius, have masses 2 kg and m kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed $5\,\mathrm{m\,s^{-1}}$ and is moving towards B at an angle of α to the line of centres, where $\cos\alpha = 0.6$. B has speed $2\,\mathrm{m\,s^{-1}}$ and is moving towards A along the line of centres (see diagram). As a result of the collision, A's loss of kinetic energy is 7.56 J, B's direction of motion is reversed and B's speed after the collision is $0.8\,\mathrm{m\,s^{-1}}$. Find

(i) the speed of A after the collision, **[3]**

(ii) the component of A's velocity after the collision, parallel to the line of centres, stating with a reason whether its direction is to the left or to the right, **[3]**

(iii) the value of m, **[3]**

(iv) the coefficient of restitution between A and B. **[2]**

7 S_A and S_B are light elastic strings. S_A has natural length 2 m and modulus of elasticity 120 N; S_B has natural length 3 m and modulus of elasticity 180 N. A particle P of mass 0.8 kg is attached to one end of each of the strings. The other ends of S_A and S_B are attached to fixed points A and B respectively, on a smooth horizontal table. The distance AB is 6 m. P is released from rest at the point of the line segment AB which is 2.9 m from A.

(i) For the subsequent motion, show that the total elastic potential energy of the strings is the same when $AP = 2.1$ m and when $AP = 2.9$ m. Deduce that neither string becomes slack. **[3]**

(ii) Find, in terms of x, an expression for the acceleration of P in the direction of AB when $AP = (2.5 + x)$ m. **[3]**

(iii) State, giving a reason, the type of motion of P and find the time taken between successive occasions when P is instantaneously at rest. **[3]**

For the instant 0.6 seconds after P is released, find

(iv) the distance travelled by P, **[3]**

(v) the speed of P. **[2]**

OCR
RECOGNISING ACHIEVEMENT

Friday 25 January 2013 – Afternoon

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

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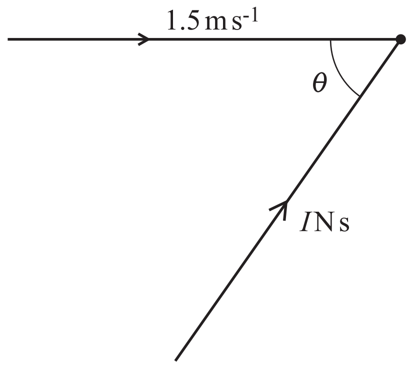
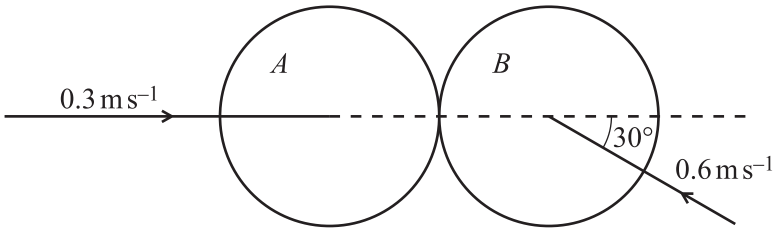
1



A ball of mass 0.6 kg is moving with speed $1.5\,\mathrm{m\,s^{-1}}$ in a straight line. It is struck by an impulse I N s acting at an acute angle θ to its direction of motion (see diagram). The impulse causes the direction of motion of the ball to change by an acute angle α, where $\sin\alpha = \frac{8}{17}$. After the impulse acts the ball is moving with a speed of $3.4\,\mathrm{m\,s^{-1}}$. Find I and θ. **[5]**

2 Two uniform smooth spheres A and B, of equal radius and equal mass, are moving towards each other on a horizontal surface. Immediately before they collide, A has speed $0.3\,\mathrm{m\,s^{-1}}$ along the line of centres and B has speed $0.6\,\mathrm{m\,s^{-1}}$ at an angle of 30° to the line of centres (see diagram).



After the collision, the direction of motion of B is at right angles to its original direction of motion. Find

(i) the speed of B after the collision, **[3]**

(ii) the speed and direction of motion of A after the collision, **[3]**

(iii) the coefficient of restitution between A and B. **[3]**

3 At time $t = 0$ s a particle P, of mass 0.3 kg, is 1 m away from a point O on a smooth horizontal plane and is moving away from O with speed $\sqrt{5}\,\mathrm{m\,s^{-1}}$. The only horizontal force acting on P has magnitude $1.5x$ N, where x is the distance OP, and acts away from O.

(i) Show that the speed of P, $v\,\mathrm{m\,s^{-1}}$, is given by $v = \sqrt{5}x$. **[4]**

(ii) Find an expression for v in terms of t. **[4]**

4 A smooth cylinder of radius a m is fixed with its axis horizontal and O is the centre of a cross-section. Particle P, of mass 0.4 kg, and particle Q, of mass 0.6 kg, are connected by a light inextensible string of length πa m. The string is held at rest with P and Q at opposite ends of the horizontal diameter of the cross-section through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v\,\mathrm{m\,s^{-1}}$ (see Fig. 2).

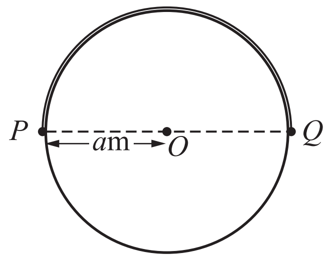


Fig. 1

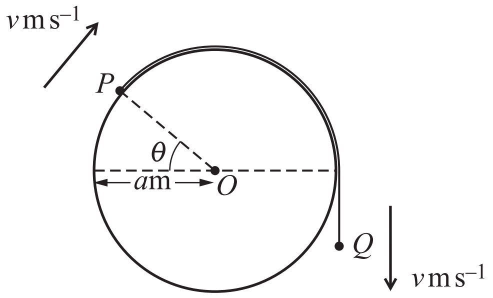


Fig. 2

(i) Show that $v^2 = 3.92a(3\theta - 2\sin\theta)$ and find an expression in terms of θ for the normal force of the cylinder on P at this time. **[9]**

(ii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $\sin\alpha = k\alpha$ where k is a constant to be found. **[2]**

5 A particle P, of mass 2.5 kg, is in equilibrium suspended from a fixed point A by a light elastic string of natural length 3 m and modulus of elasticity 36.75 N. Another particle Q, of mass 1 kg, is released from rest at A and falls freely until it reaches P and becomes attached to it.

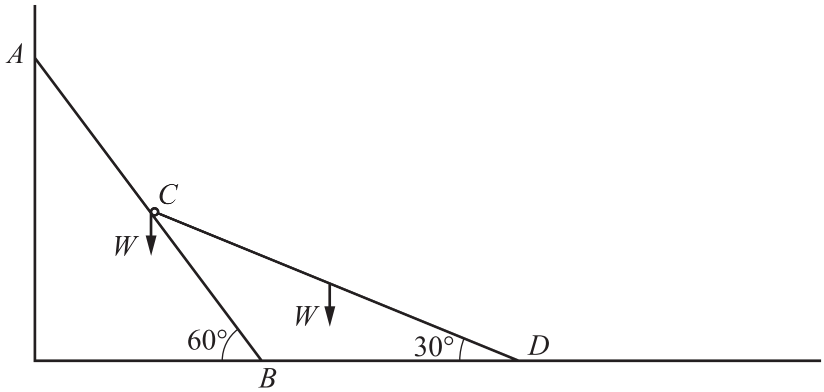
(i) Show that the speed of the combined particles, immediately after Q becomes attached to P, is $2\sqrt{2}\,\mathrm{m\,s^{-1}}$. **[6]**

The combined particles fall a further distance X m before coming to instantaneous rest.

(ii) Find a quadratic equation satisfied by X, and show that it simplifies to $35X^2 - 56X - 80 = 0$. **[6]**

[Questions 6 and 7 are printed overleaf]

6 A uniform rod AB, of weight W and length $2l$ is in equilibrium at 60° to the horizontal with A resting against a smooth vertical plane and B resting on a rough section of a horizontal plane. Another uniform rod CD, of length $\sqrt{3}l$ and weight W, is freely jointed to the mid-point of AB at C; its other end D rests on a smooth section of the horizontal plane. CD is inclined at 30° to the horizontal (see diagram).



(i) Show that the force exerted by the horizontal plane on CD is $\frac{1}{2}W$. Find the normal component of the force exerted by the horizontal plane on AB. **[5]**

(ii) Find the magnitude and direction of the force exerted by CD on AB. **[3]**

(iii) Given that AB is in limiting equilibrium, find the coefficient of friction between AB and the horizontal plane. **[5]**

7 A simple pendulum consists of a light inextensible string of length 0.8 m and a particle P of mass m kg. The pendulum is hanging vertically at rest from a fixed point O when P is given a horizontal velocity of $0.3\,\mathrm{m\,s^{-1}}$.

(i) Show that, in the subsequent motion, the maximum angle between the string and the downward vertical is 0.107 radians, correct to 3 significant figures. **[3]**

(ii) Show that the motion may be modelled as simple harmonic motion, and find the period of this motion. **[5]**

(iii) Find the time after the start of the motion when the velocity of the particle is first $-0.2\,\mathrm{m\,s^{-1}}$ and find the angular displacement of OP from the downward vertical at this time. **[6]**

OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4733420613

INSTRUCTIONS TO CANDIDATES

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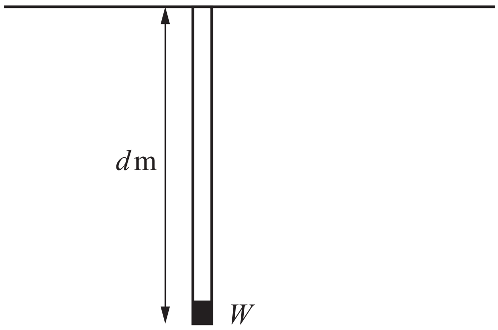
1



A small object W of weight 100 N is attached to one end of each of two parallel light elastic strings. One string is of natural length 0.4 m and has modulus of elasticity 20 N; the other string is of natural length 0.6 m and has modulus of elasticity 30 N. The upper ends of both strings are attached to a horizontal ceiling and W hangs in equilibrium at a distance d m below the ceiling (see diagram). Find d. **[5]**

2 A particle of mass 0.3 kg is projected horizontally under gravity with velocity 3.5 m s^{-1} from a point 0.4 m above a smooth horizontal plane. The particle first hits the plane at point A; it bounces and hits the plane a second time at point B. The distance AB is 1 m. Calculate

(i) the vertical component of the velocity of the particle when it arrives at A, and the time taken for the particle to travel from A to B, **[3]**

(ii) the coefficient of restitution between the particle and the plane, **[3]**

(iii) the impulse exerted by the plane on the particle at A. **[2]**

3 A particle P of mass 0.2 kg moves on a smooth horizontal plane. Initially it is projected with velocity 0.8 m s^{-1} from a fixed point O towards another fixed point A. At time t s after projection, P is x m from O and is moving with velocity v m s^{-1}, with the direction OA being positive. A force of $(1.5t - 1)$ N acts on P in the direction parallel to OA.

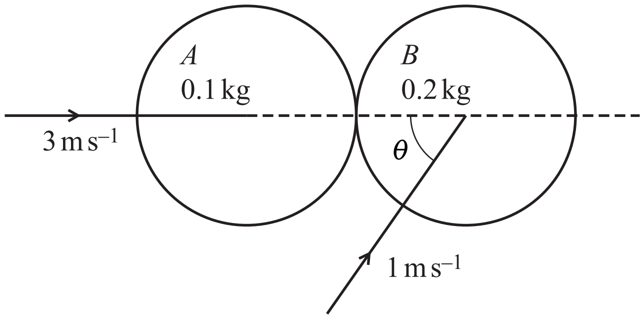
(i) Find an expression for v in terms of t. **[3]**

(ii) Find the time when the velocity of P is next 0.8 m s^{-1}. **[2]**

(iii) Find the times when P subsequently passes through O. **[4]**

(iv) Find the distance P travels in the third second of its motion. **[2]**

4 Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.1 kg and B has mass 0.2 kg. Immediately before the collision A is moving with speed $3\,\mathrm{m\,s^{-1}}$ along the line of centres, and B is moving away from A with speed $1\,\mathrm{m\,s^{-1}}$ at an acute angle θ to the line of centres, where $\cos\theta = 0.6$ (see diagram).

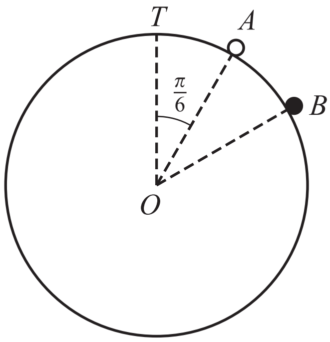


The coefficient of restitution between the spheres is 0.8. Find

(i) the velocity of A immediately after the collision, **[6]**

(ii) the angle turned through by the direction of motion of B as a result of the collision. **[5]**

5



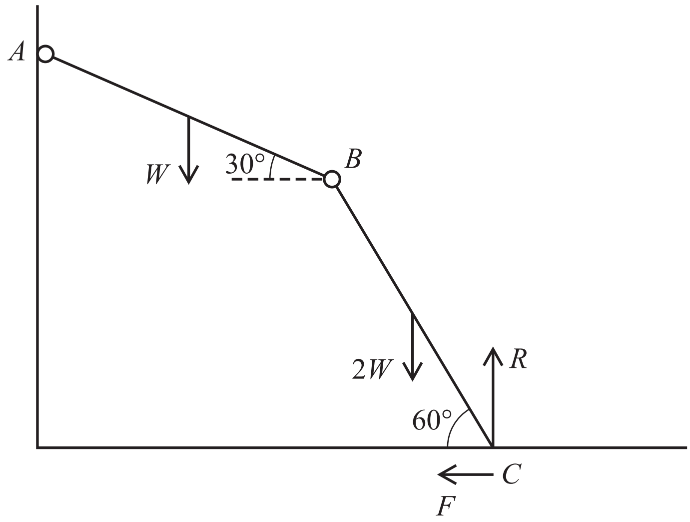
A fixed smooth sphere of radius 0.6 m has centre O and highest point T. A particle of mass m kg is released from rest at a point A on the sphere, such that angle TOA is $\frac{\pi}{6}$ radians. The particle leaves the surface of the sphere at B (see diagram).

(i) Show that $\cos TOB = \frac{\sqrt{3}}{3}$. **[6]**

(ii) Find the speed of the particle at B. **[2]**

(iii) Find the transverse acceleration of the particle at B. **[2]**

6 Two uniform rods AB and BC, each of length $2l$, are freely jointed at B. The weight of AB is W and the weight of BC is $2W$. The rods are in a vertical plane with A freely pivoted at a fixed point and C resting in equilibrium on a rough horizontal plane. The normal and frictional components of the force acting on BC at C are R and F respectively. The rod AB makes an angle 30° to the horizontal and the rod BC makes an angle 60° to the horizontal (see diagram).



(i) By considering the equilibrium of rod BC, show that $W + \sqrt{3}F = R$. **[2]**

(ii) By taking moments about A for the equilibrium of the whole system, find another equation involving W, F and R. **[4]**

(iii) Given that the friction at C is limiting, calculate the value of the coefficient of friction at C. **[5]**

7 A particle P of mass m kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity $39.2m$ N. The other end of the string is attached to a fixed point O. The particle is released from rest at O.

(i) Show that, while the string is in tension, the particle performs simple harmonic motion about a point 1 m below O. **[3]**

(ii) Show that when P is at its lowest point the extension of the string is 0.8 m. **[3]**

(iii) Find the time after its release that P first reaches its lowest point. **[6]**

(iv) Find the velocity of P 0.8 s after it is released from O. **[4]**

Friday 6 June 2014 – Afternoon

A2 GCE MATHEMATICS

4730/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle P of mass 0.3 kg is moving on a smooth horizontal surface with speed $0.8\,\text{m}\,\text{s}^{-1}$ when it is struck by a horizontal impulse. The magnitude of the impulse is 0.6 N s.

(i) (a) Find the greatest possible speed of P after the impulse acts.

(b) Find the least possible speed of P after the impulse acts. **[3]**

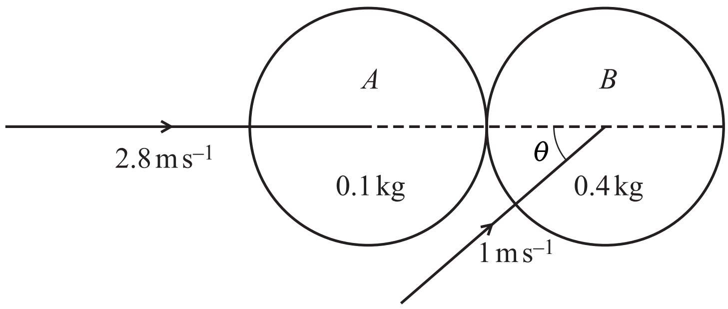
(ii) In fact the speed of P after the impulse acts is $2.5\,\text{m}\,\text{s}^{-1}$. Find the angle the impulse makes with the original direction of travel of P and draw a sketch to make this direction clear. **[4]**

2 One end of a light elastic string, of natural length 0.6 m and modulus of elasticity 30 N, is attached to a fixed point O. A particle P of weight 48 N is attached to the other end of the string. P is released from rest at a point d m vertically below O. Subsequently P just reaches O.

(i) Find d. **[4]**

(ii) Find the magnitude and direction of the acceleration of P when it has travelled 1.3 m from its point of release. **[4]**

3



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.1 kg and B has mass 0.4 kg. Immediately before the collision A is moving with speed $2.8\,\text{m}\,\text{s}^{-1}$ along the line of centres, and B is moving with speed $1\,\text{m}\,\text{s}^{-1}$ at an angle θ to the line of centres, where $\cos\theta = 0.8$ (see diagram). Immediately after the collision A is stationary. Find

(i) the coefficient of restitution between A and B, **[5]**

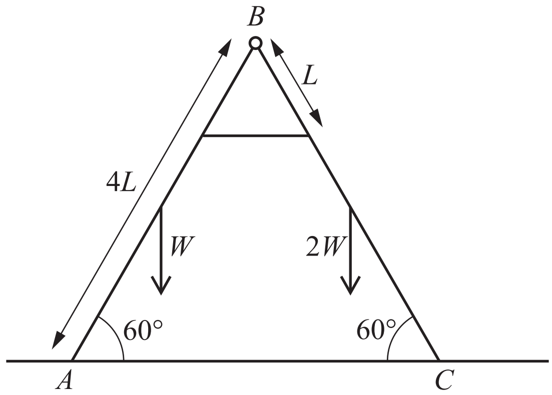
(ii) the angle turned through by the direction of motion of B as a result of the collision. **[4]**

4 A particle P of mass 0.4 kg is projected horizontally with speed $2\,\mathrm{m\,s^{-1}}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving away from O with speed $v\,\mathrm{m\,s^{-1}}$. There is a force of magnitude $1.6v^2$ N resisting the motion of P.

(i) Find an expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ in terms of v, and hence show that $v = 2\mathrm{e}^{-4x}$. **[5]**

(ii) Find the distance travelled by P in the 0.5 seconds after it leaves O. **[5]**

5



Two uniform rods AB and BC, each of length $4L$, are freely jointed at B, and rest in a vertical plane with A and C on a smooth horizontal surface. The weight of AB is W and the weight of BC is $2W$. The rods are joined by a horizontal light inextensible string fixed to each rod at a point distance L from B, so that each rod is inclined at an angle of 60° to the horizontal (see diagram).

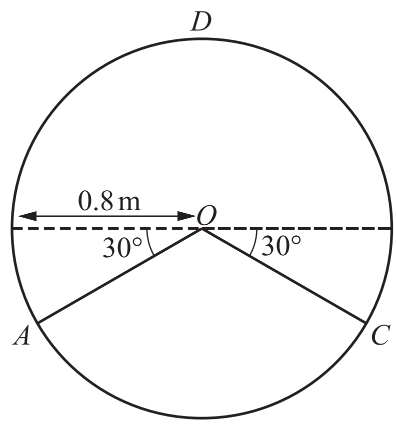
(i) By considering the equilibrium of the whole body, show that the force acting on BC at C is $1.75W$ and find the force acting on AB at A. **[4]**

(ii) Find the tension in the string in terms of W. **[4]**

(iii) Find the horizontal and vertical components of the force acting on AB at B, and state the direction of the component in each case. **[3]**

Question 6 begins on page 4.

6

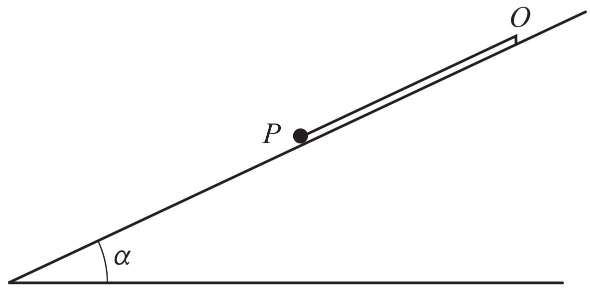


A hollow cylinder is fixed with its axis horizontal. O is the centre of a vertical cross-section of the cylinder and D is the highest point on the cross-section. A and C are points on the circumference of the cross-section such that AO and CO are both inclined at an angle of 30° below the horizontal diameter through O. The inner surface of the cylinder is smooth and has radius 0.8 m (see diagram). A particle P, of mass m kg, and a particle Q, of mass $5m$ kg, are simultaneously released from rest from A and C, respectively, inside the cylinder. P and Q collide; the coefficient of restitution between them is 0.95.

(i) Show that, immediately after the collision, P moves with speed $6.3\,\mathrm{m\,s^{-1}}$, and find the speed and direction of motion of Q. **[8]**

(ii) Find, in terms of m, an expression for the normal reaction acting on P when it subsequently passes through D. **[6]**

7



One end of a light elastic string, of natural length 0.3 m, is attached to a fixed point O on a smooth plane that is inclined at an angle α to the horizontal, where $\sin\alpha = 0.2$. A particle P of mass m kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane and has modulus of elasticity $2.45m$ N (see diagram).

(i) Show that in the equilibrium position the extension of the string is 0.24 m. **[2]**

P is given a velocity of $0.3\,\mathrm{m\,s^{-1}}$ down the plane from the equilibrium position.

(ii) Show that P performs simple harmonic motion with period 2.20 s (correct to 3 significant figures), and find the amplitude of the motion. **[6]**

(iii) Find the distance of P from O and the velocity of P at the instant 1.5 seconds after P is set in motion. **[5]**